CDS 112: Winter 2014/2015

Homework #6: due Wednesday, March 11, 2015

Problem #1: (20 points)

In this problem, you will revisit the "Kidnapped Robot" problem in last week's homework. Fixed lag smoothing can improve the estimate of vehicle position at the cost of delay in making that information available. For the same system GPS and accelerometer measurement system described in the previous Problem set, find the fixed lag smoothing equations for a smoother which delays the position estimate by 4 seconds (e.g., by two delays in the GPS measurement cycle). What is the covariance improvement of the fixed lag smoother over that of the Kalman filter?

Problem 2:(20 points) The goal of preserving fuel use is critical for spacecraft design and deployment. In this problem you will consider a highly simplified version of a satellite attitude control problem. Assume a single rigid body satellite free-floating above earth is constrained to move in a plane. In this simplified model, θ is the angle which describes the satellite's orientation in that plane. The dynamics which relate the control input, u to the satellite's orientation are:

$$I\ddot{\theta} \;=\; u \tag{1}$$

where I is the rotational inertia of the satellite, and u can be interpreted as the torque applied to the satellite. In this problem, the torque is provided by a *thruster*. For example, a compressed gas canister can provide thrust, and the loss of gas is roughly proportional to the amount of thrust generated. Note that when the gas is all used up, reorientation is no longer possible, and so in that case, it is essential to minimize gas usage by minimizing the thrust used to carry out a maneuver. As an alternative to a gas canister, one could use an *ion thruster*, whose energy can be continually replenished by a solar panel. In this case, the use of the thruster is still costly, but it must be balanced against performance.

Assume that the spacecraft must carry out a reorientation maneuver, starting from an initial orientation θ_0 at time $t_0 = 0$, and ending at a final orientation, θ_f.

Part (a): LQG (15 points) Design a **steady-state** LQG controller, assuming that the cost is:

$$J = E\left\{\int_0^\infty [x^T Q x + u^T R u]dt\right\}$$

where the state[1] x consists of:

$$x = \begin{bmatrix}(\theta - \theta_f)\\ \dot{\theta}\end{bmatrix}$$

where the weighting matrices Q and R take the form:

$$Q = \begin{bmatrix}10 & 0\\ 0 & 1\end{bmatrix} \qquad R = 1$$

[1]Note that the state can be written simply as $x = [\theta\ \dot{\theta}]^T$ if the θ-coordinates are adjusted so that $\theta_f = 0$.

and where a small disturbance due to a *gravity gradient* acts on the system in the following way

$$I\ddot{\theta} \;=\; u + \eta(t) \tag{2}$$

where $\eta(t)$ can be modeled by zero mean white Gaussian noise with covariance 0.001. Assume that only satellite orientation measurements are available for the estimator:

$$y(t) = \theta(t) + \omega(t)$$

and assume the measurement noise $\omega(t)$ is zero mean white Gaussiam with 0.5 degrees^2 variance.

Note: this problem is given in a continous time fashion. If you'd prefer to design both controller and estimator (or just estimator) in discrete time, that is fine.

Part(b): (5 points) Compute the closed loop poles of the controller, and also the closed loop poles of the estimator.