ME 115(b): Final Exam, Spring 2005-06

Instructions

1. Limit your total time to 5 hours. That is, it is okay to take a break in the middle of the exam if you need to ask me a question, or go to dinner, etc.
2. You may use any class notes, books, or other written material.
3. You may use mathematica or any software or computational tools to assist you.
4. Feel free to ask me or the T.A.s questions about the exam.
5. The final is due by 5:00 p.m. on the last day of the final period. If you need your grade turned in to the registrar for purposes of graduation, then the final is due at 5:00 p.m. on Wednesday, May 31.
6. The point values are listed for each problem to assist you in allocation of your time.
7. Please put all of your answers in a blue book, or carefully staple your work together in the proper order.

Problem #1: (18 Points)

Consider the planar mechanism shown in Figure 1. This is a "geared" 6-bar mechanism which I have recently used as a leg thrusting mechanism for a "hopping" robot. Note that the spring is inconsequential for your analysis. The gears in this mechanism have the same radius. That is, as one gear in a gear pair turns, the other gear rotates at the same angular velocity, but opposite sense of direction.

1. What is the mobility of this mechanism?
2. Sketch the form of the structure/velocity equations (by "sketch", I mean that I don't necessarily need the exact algebraic terms of every term in the structure equation, but I do want to see the basic form of the equation. However, I'm happy to take the full structure equation if you like!).
3. Physically describe the conditions under which this mechanism becomes singular, and interpret it in light of the above derivations.

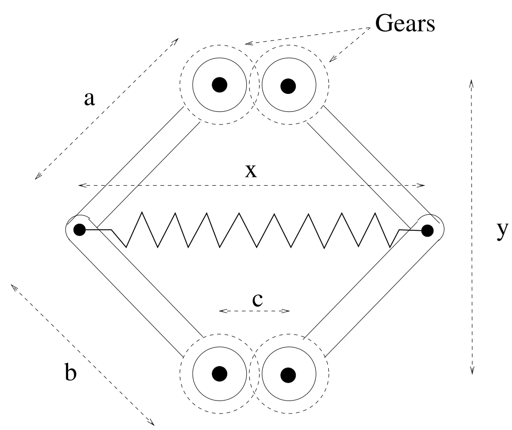

Figure 1: Planar 6-bar Geared Mechanism

Problem #2 (17 Points)

In class we discussed a variety of "redundancy resolution" techniques. One method which we didn't discuss is the "Augmented Jacobian" technique. In this method, one "augments" the "task vector" by identifying additional "tasks" so that the resulting systems is no longer redundant. For example, let $\vec{x}$ be the location of the end-effector. $\vec{x}$ is related to the manipulator joint angles through the forward kinematics relationship:

$$\vec{x} = f(\vec{\theta})$$

where $\vec{\theta}$ is an n-vector of joint angles and $\vec{x}$ represents the p independent end-effector coordinates. One can define $(n - p)$ additional "tasks":

$$\vec{\phi} = g(\vec{\theta})$$

where $\vec{\phi}$ is the $(n-p)\times 1$ "augmented task vector." For example, one might define elements of ϕ as the orientation or elevation of interior links. Using the augmented task vector, one can then define the "Augmented Jacobian" through the relationship:

$$\begin{bmatrix} \dot{\vec{x}} \\ \dot{\vec{\phi}} \end{bmatrix} = \begin{bmatrix} \mathbf{J} \\ \frac{\partial g(\vec{\theta})}{\partial \vec{\theta}} \end{bmatrix}$$

(a) Describe the conditions under which the Augmented Jacobian loses rank.

(b) Physically interpret each of the conditions in Part (a).

Problem #3 (20 Points): Special Configurations of "Slider-Crank" linkages.

Consider the "slider-crank" four bar linkage shown Fig. 2. This mechanism, which is commonly used as the piston mechanism in an internal combustion engine, consists of three revolute joints and one prismatic joint. The joints are numbered successively, with the first joint being at the left of the figure, and the fourth joint being the prismatic joint.

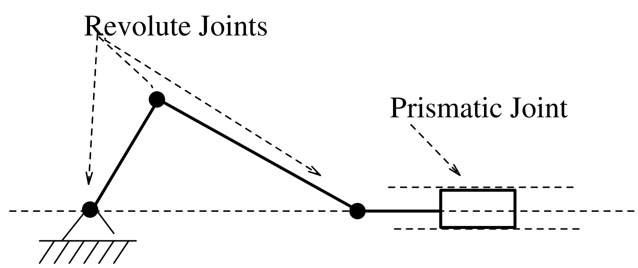

Figure 2: Slider Crank Mechanism

Part (a): One of the special configurations is obvious: the piston (or prismatic joint) is at "top dead center," and the cylinder comes momentarily to rest in this position. Show that this configuration is indeed a special configuration. In particular, show that joint 4 has a stationary configuration when $\theta_1 = 0$ and $\theta_2 = 0$.

Part (b): Develop an expression for the stationary configurations of joint 1. What are the necessary conditions for joint 1 to have a stationary configuration?

Problem #4 (10 Points): parallel mechanisms

Figure 3(a) shows a planar "crossed" parallel mechanism. It consists of three active prismatic joints and six passive revolute joints.

Part (c): Find and sketch the kinematic and actuator singular configurations of this mechanism.

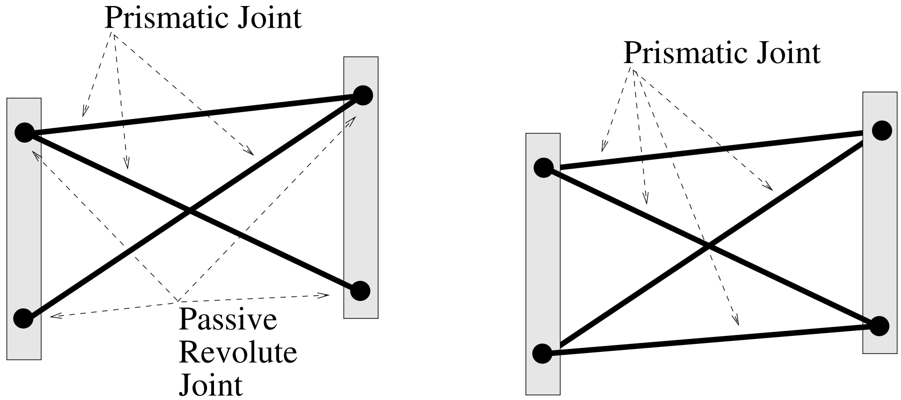

Figure 3: (a) planar parallel mechanism with 3 active prismatic joints; (b) planar parallel mechanism with 4 active prismatic joints.

Figure 3(b) is similar to Figure 3(a), except that it has an extra (or fourth) prismatic joint. Such a mechanism is considered to be "overconstrained."

Part (b): Can you describe some possible advantages and disadvantages that arise from overconstraint. What are the singular configurations of this mechanism?

Problem #5 (15 Points): (force closure)

A 3-dimensional body (such as an ellipsoid) is grasped by two fingers in an antipodal point grasp (Figure 4). Let one of the contacts be modelled by the point contact with friction model. Let the other contact be modelled by a soft finger contact.

Part (a): Sketch the structure of the grasp map for this grasp

Part (b): Is this grasp force closure? Justify your answer using one of the force closure definitions.

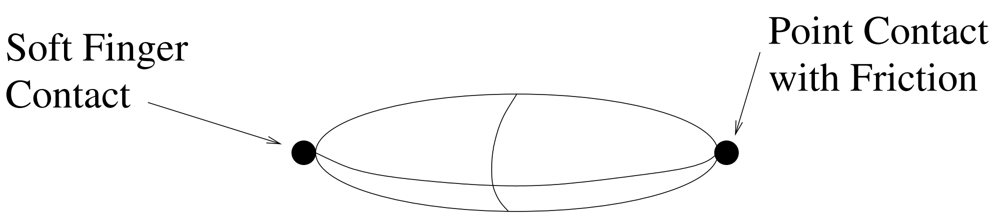

Figure 4: 2-fingered grasp of 3-dimensional Object

Problem #6 (20 Points): (Grasp and Rolling Contact)

One of the main motivations to study the *contact equations* was the realization that real fingertips and grasped objects do not maintain a fixed contact point while they interact in the context of a grasp. In class (and in the MLS text), we derived the following kinematic equations for a grasp involving multiple fingers (where each finger was a serial chain linkage, with joint variables $\vec{\theta}_i = \{\theta_{i1}, \theta_{i2}, \ldots, \theta_{in_i}\}$ for the i^{th} finger):

$$G^T \; V_{PO}^b = J_H(\vec{\theta})\dot{\vec{\theta}} \tag{1}$$

where G is the grasp map (assuming a fixed point of contact between the finger and the object), V_{PO} is the velocity of the grasped object (as measured with respect to the "palm" frame), J_H is the hand Jacobian, and $\vec{\theta} = [\vec{\theta}_1^T \; \vec{\theta}_2^T \; \ldots \; \vec{\theta}_N^T]$ is the vector of all joint angles.

Equation (1) was derived assuming a "point-like" finger tip which did not roll or slide as the object is manipulated. Now assume that the finger tip is a 3-dimensional surface (or a curve in the case of planar fingers and object). How does Equation (1) change when the point-finger assumption is replaced by the assumption of a curved fingertip that can roll on the surface of the object?

If it simplifies your discussion and/or analysis, you can analyze the simpler case of planar fingers and objects.