ME 115(b): Final Exam, Spring 2009-10

Instructions

1. Limit your total time to 5 hours. That is, it is okay to take a break in the middle of the exam if you need to ask me a question, or go to dinner, etc.
2. You may use any class notes, books, or other written material.
3. You may use Mathematica, MATLAB, or any software or computational tools to assist you.
4. Feel free to ask me or the T.A.s questions about the exam.
5. The final is due by 5:00 p.m. on the last day of the final period.
6. The point values are listed for each problem to assist you in allocation of your time.
7. Please staple your work together in the proper order, or put your answers and calculations in a blue book.

Problem #1: (20 Points)

Consider the planar mechanism shown in Figure 1. This is a "geared" 6-bar mechanism, which can be used to store and release energy. Note that the spring is inconsequential for your analysis. The gears in this mechanism have the same radius. That is, as one gear in a gear pair turns, the other gear rotates at the same angular velocity, but opposite sense of direction.

1. What is the mobility of this mechanism?
2. Sketch the form of the structure/velocity equations (by "sketch", I mean that I don't necessarily need the exact algebraic terms of every term in the structure equation, but I do want to see the basic form of the equation. However, I'm happy to take the full structure equation if you like!).
3. Physically describe the conditions under which this mechanism becomes singular, and interpret it in light of the above derivations.

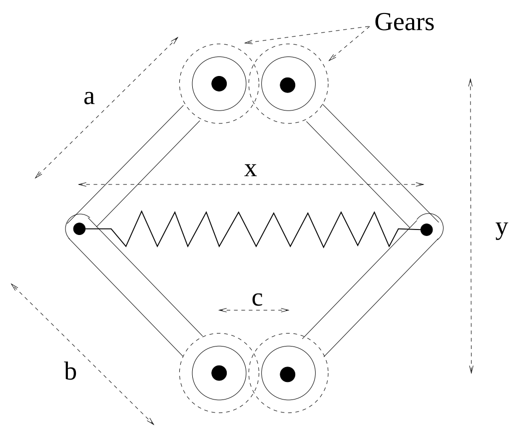

Figure 1: Planar 6-bar Geared Mechanism

Problem #2 (17 Points)

In class we discussed a variety of "redundancy resolution" techniques. One method which we didn't discuss is the "Augmented Jacobian" technique. In this method, one "augments" the "task vector" by identifying additional "tasks" so that the resulting systems is no longer redundant. For example, let $\vec{x}$ be the location of the end-effector. $\vec{x}$ is related to the manipulator joint angles through the forward kinematics relationship:

$$\vec{x} = f(\vec{\theta})$$

where $\vec{\theta}$ is an n-vector of joint angles and $\vec{x}$ represents the p independent end-effector coordinates. One can define $(n - p)$ additional "tasks":

$$\vec{\phi} = g(\vec{\theta})$$

where $\vec{\phi}$ is the $(n-p)\times 1$ "augmented task vector." For example, one might define elements of ϕ as the orientation or elevation of interior links. Using the augmented task vector, one can then define the "Augmented Jacobian" through the relationship:

$$\begin{bmatrix} \dot{\vec{x}} \\ \dot{\vec{\phi}} \end{bmatrix} = \begin{bmatrix} \mathbf{J} \\ \frac{\partial g(\vec{\theta})}{\partial \vec{\theta}} \end{bmatrix}$$

(a) Describe the conditions under which the Augmented Jacobian loses rank.

(b) Physically interpret each of the conditions in Part (a).

Problem #3 (15 Points): parallel mechanisms

Figure 2(a) shows a planar "crossed" parallel mechanism. It consists of three active prismatic joints and six passive revolute joints. Figure 2(b) is similar to Figure 2(a), except that it has an extra (or fourth) prismatic joint. Such a mechanism is considered to be "overconstrained."

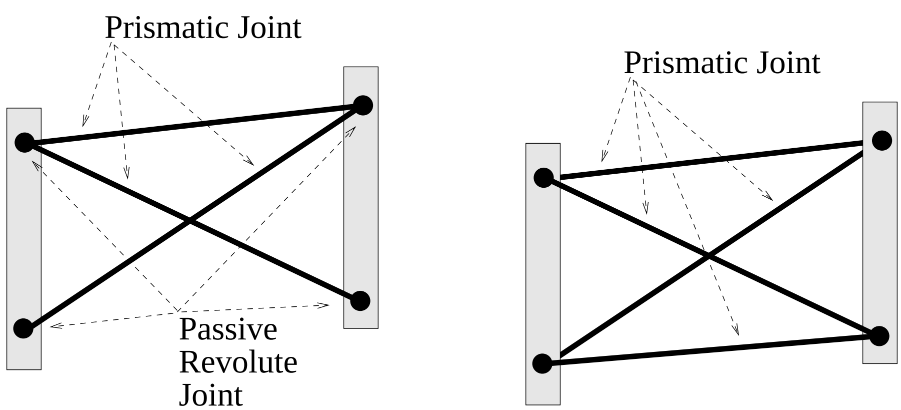

Figure 2: (a) planar parallel mechanism with 3 active prismatic joints; (b) planar parallel mechanism with 4 active prismatic joints.

Part (a): Find and sketch the kinematic and actuator singular configurations of this mechanism.

Part (b): Can you describe some possible advantages and disadvantages that arise from overconstraint. What are the singular configurations of this mechanism?

Problem #5 (20 Points): manipulator Jacobian and singularities.

Consider the 3-jointed RPR manipulator in Figure 3. Assume that the first axis is vertical. The second (prismatic) axis is orthogonally intersects the first axis. The third axis orthogonally intersects the prismatic joint axis, and is simultaneously orthogonal to the first axis. We are only concerned with position the origin of the end-effector frame.

Compute the hybrid Jacobian matrix for this manipulator.

Describe its singular configurations.

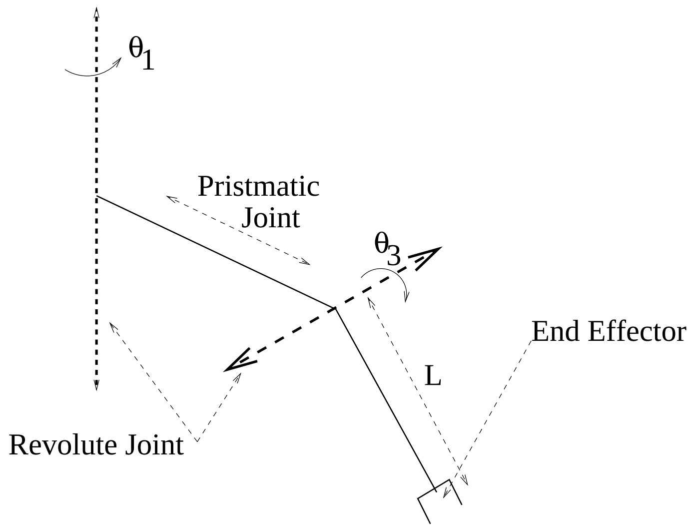

Figure 3: RPR manipulator