ME 115(b): Final Exam, Spring 2011-12

Instructions

1. Limit your total time to 5 hours. That is, it is okay to take a break in the middle of the exam if you need to ask me a question, or go to dinner, etc.
2. You may use any class notes, books, or other written material.
3. You may use mathematica or any software or computational tools to assist you.
4. Feel free to ask me or the T.A.s questions about the exam.
5. The final is due by 5:00 p.m. on the last day of the final period. If you need your grade turned in to the registrar for purposes of graduation, then the final is due at 5:00 p.m. on Friday, June 8.
6. The point values are listed for each problem to assist you in allocation of your time.
7. Please put all of your answers in a blue book, or carefully staple the pages of your work together in the proper order (adding page numbers and/or problem numbers to the pages will help me).

Problem #1: (20 Points)

Consider the planar mechanism shown in Figure 1. This is a "geared" 6-bar mechanism which I have used in the past as a leg thrusting mechanism for a "hopping" robot. Note that the spring is inconsequential to your analysis. All of the gears in this mechanism have the same radius (or "pitch circle"). That is, as one gear in a gear pair turns, the other gear rotates at the same angular velocity, but opposite sense of direction.

1. What is the mobility of this mechanism? Back up your answer with some analysis.
2. Sketch the form of the structure/velocity equations (by "sketch", I mean that I don't necessarily need the exact algebraic terms of every term in the structure equation, but I do want to see the basic form of the equation. However, I'm happy to take the full structure equation if you like!).
3. Physically describe the special configuration of this mechanism, if any, and interpret them in light of the above derivations.

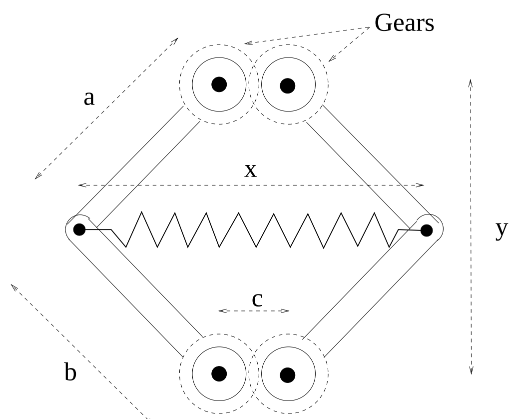

Figure 1: Planar 6-bar Geared Mechanism

Problem #2 (15 Points): (force closure)

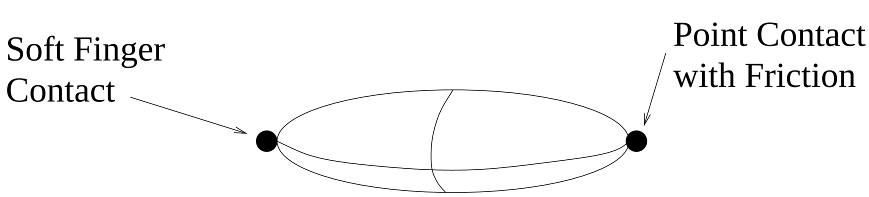

Figure 2: 2-fingered grasp of 3-dimensional Object

A 3-dimensional body (such as an ellipsoid) is grasped by two fingers in an antipodal point grasp (Figure 2). Let one of the contacts be modelled by the point contact with friction model. Let the other contact be modelled by a soft finger contact.

Part (a): Construct the grasp map for this grasp

Part (b): Is this grasp force closure? Justify your answer using one of the force closure definitions.

Problem #3 (20 Points): (Grasp and Rolling Contact)

One of the main motivations to study the *contact equations* was the realization that real fingertips and grasped objects do not maintain a fixed contact point while they interact in the context of a grasp. In class (and in the MLS text), we derived the following kinematic equations for a grasp involving multiple fingers (where each finger was a serial chain linkage, with joint variables $\vec{\theta_i} = \{\theta_{i1}, \theta_{i2}, \ldots, \theta_{in_i}\}$ for the i^{th} finger):

$$G^T \ V_{PO}^b = J_H(\vec{\theta})\dot{\vec{\theta}} \tag{1}$$

where G is the grasp map (assuming a fixed point of contact between the finger and the object), V_{PO} is the velocity of the grasped object (as measured with respect to the "palm" frame), J_H is the hand Jacobian, and $\vec{\theta} = [\vec{\theta}_1^T \ \vec{\theta}_2^T \ \ldots \ \vec{\theta}_N^T]$ is the vector of all joint angles.

Equation (1) was derived assuming a "point-like" finger tip which did not roll or slide as the object is manipulated. Now assume that the finger tip is a 3-dimensional surface (or a curve in the case of planar fingers and object). How does Equation (1) change when the point-finger assumption is replaced by the assumption of a curved fingertip that can roll on the surface of the object?

If it simplifies your discussion and/or analysis, you can analyze the simpler case of planar fingers and objects.

Problem #4 (20 Points): manipulator Jacobian and singularities.

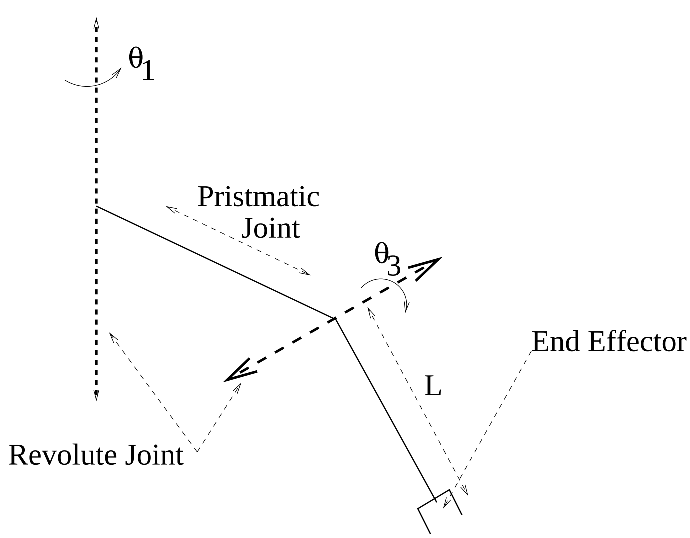

Figure 3: RPR manipulator

Consider the 3-jointed RPR manipulator in Figure 3. Assume that the first axis is vertical. The second (prismatic) axis is orthogonally intersects the first axis. The third axis orthogonally intersects the prismatic joint axis, and is simultaneously orthogonal to the first axis. We are only concerned with position the origin of the end-effector frame.

Compute the spatial Jacobian matrix for this manipulator.

Describe its singular configurations.

Problem #5**:** (30 points)

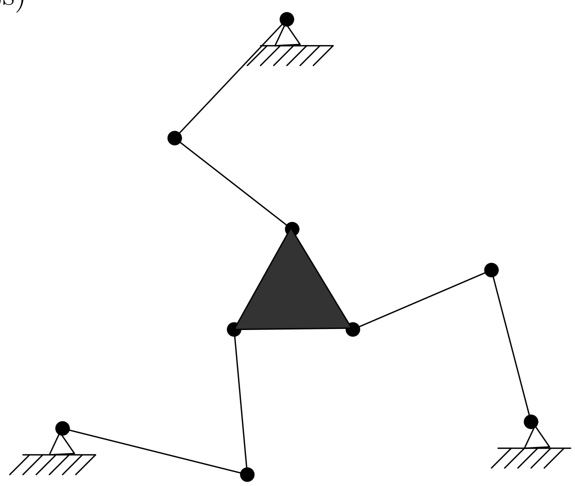

Figure 4: Planar Mechanism

Consider the planar mechanism shown in Figure 4.

1. What is the mobility of this mechanism?
2. Derive the structure equations.
3. Derive the velocity equations.
4. Physically interpret (using your answer to the last question as a guide if necessary) the conditions under which this mechanism becomes singular.

Extra Credit #4 (10 Points): differential geometry of curves

Let $\alpha\colon I \to \mathbb{R}^2$ be a regular parametrized plane curve. The curve:

$$\beta(s) = \alpha(s) + \frac{1}{\kappa(s)}\vec{n}(s)$$

is known as the *evolute* of $\alpha(t)$. In this formula, $\kappa(s)$ is the curvature at curve parameter value s, while $\vec{n}$ is the unit normal vector at s. Assume that s is an arc-length parameter. Evolute curves are important in gear theory.

(a) Show that the tangent at s of the evolute of $\alpha(s)$ is the normal to α at s.

(b) what is the evolute of a circle?