# ME 115(a): Final Exam, 2000/2001

## Instructions

1. Limit your total time to 5 hours. That is, it is okay to take a break in the middle of the exam if you need to ask me a question, or go to dinner, etc.
2. You may use any class notes, books, or other written material. You may not discuss this final with other students or other people except me or the T.A.s.
3. You may use mathematica or any software or computational tools to assist you. However, if you find that solution approach requires a lot of algebra or a lot of computation, then you are probably taking a less than optimal approach.
4. The final is due by 5:00 p.m. on the last day of finals.
5. The point values are listed for each problem to assist you in allocation of your time.

**Problem 1:** (35 Points)

Consider the three jointed manipulator shown in Figure 1. This first joint of this manipulator is a prismatic joint, while the next two joints are revolute joints that intersect orthogonally to each other, and orthogonally to the prismatic joint axis.

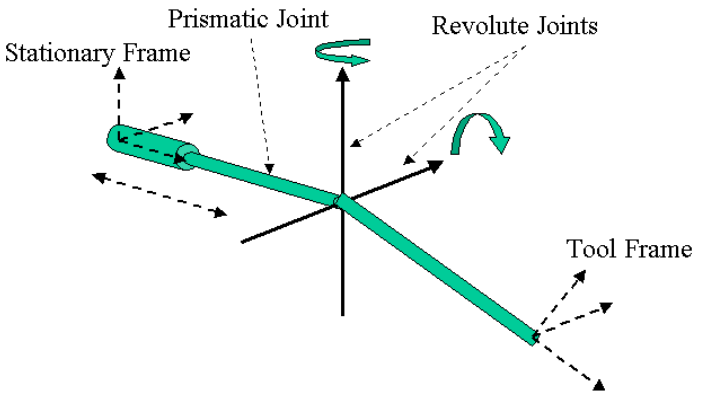


Figure 1: Schematic of PRR Manipulator Geometry

**Part (a):** (3 points) Determine the Denavit-Hartenberg parameters.

**Part (b):** (7 points) Using either the Denavit-hartenberg approach or the product-of-exponentials approach, determine the forward kinematics. That is, relate the coordinates of the origin of the tool frame to the joint angles.

**Part (c):** (15 points) Solve the inverse kinematics of this manipulator, assuming that the goal is to position the origin of the tool frame.

**Part (d):** (10 points) Find the spatial Jacobian matrix of this manipulator.

**Problem 2:** (15 points)

We discovered numerous ways to represent and manipulate spatial displacements. Those crazy kinematicians have yet another variation on the same theme using something called "dual numbers." A dual number, $\tilde{a}$, takes the form:

$$\tilde{a} = a_r + \epsilon \, a_d$$

where $a_r$ is the "real" part of the dual number and $a_d$ is the "dual" or "pure" part of the dual number. The bases for the dual numbers are 1 and $\epsilon$, and they obey the rules:

$$\begin{aligned} 1 \cdot 1 &= 1 \\ 1 \cdot \epsilon &= \epsilon \cdot 1 = \epsilon \\ \epsilon^2 &= 0 \end{aligned}$$

Dual numbers have many interesting properties, though we will only explore one aspect of their characteristics in this problem.

**Part (a):** (10 points). We can represent spatial displacments as "dual rotation matrices." That is, if a spatial displacement has the form:

$$g = \begin{bmatrix} R & \overline{p} \\ \overline{0}^T & 1 \end{bmatrix}$$

where $R \in SO(3)$ and $\overline{p} \in \mathbb{R}^3$, then the dual representation of the spatial displacement is:

$$\tilde{g} = R + \epsilon(\hat{p}R)$$

1. Show that $\tilde{g}$ is an orthogonal matrix.
2. If $g_1$ and $g_2$ are spatial displacements, and $\tilde{g}_1$ and $\tilde{g}_2$ there dual equivalents, then show that $g_1\ g_2$ and $\tilde{g}_1\ \tilde{g}_2$ are equivalent.

*Hint*: in some ways of solving this problem, it might be useful to recall that if $A \in SO(3)$ and $\overline{v} \in \mathbb{R}^3$, then $\widehat{(A\overline{v})} = A\hat{v}A^T$).

**Part (b):** (5 Points). We can also use dual numbers to represent twist coordinates. Let $\xi = [\overline{V}, \overline{\omega}]^T$ be a vector twist coordinates. Its dual representation is $\tilde{\xi} = \overline{\omega} + \epsilon\overline{V}$. Show that

1. if $g$ is a spatial displacement, and $\xi$ is a twist, then $Ad_g\xi$ is equivalent to $\tilde{g}\tilde{\xi}$.
2. If $\xi_1$ and $\xi_2$ are two twists, then the dual part of dual dot product $\tilde{\xi}_1 \cdot \tilde{\xi}_2$ is equivalent to the reciprocal product of $\xi_1$ and $\xi_2$. (Note, the real part of this product is called the "Klein product.").

**Problem 3:** (10 points) Do Problem 8 in chapter 3 of MLS (page 149).

**Problem 4:** (20 points) Problem 18(a,b) in Chapter 3 of MLS (page 78). Part (a) counts for 10 points, and part (b) counts for 10 points.

**Problem 5:** (20 Points) Do problem 11(d,e) in Chapter 2 of the MLS text. Recall that you have already done 11(a,b) in Homework #3. You need not rederive any results that were already developed in this homework solution.