ME 115(b): Final Exam, Spring 2000/2001

Instructions

1. Limit your total time to 5 hours. That is, it is okay to take a break in the middle of the exam if you need to ask me a question, or go to dinner, etc.
2. You may use any class notes, books, or other written material.
3. You may use mathematica or any software or computational tools to assist you.
4. Feel free to ask me or the T.A.s questions about the exam.
5. The final is due by 5:00 p.m. on the last day of the final period.
6. The point values are listed for each problem to assist you in allocation of your time.
7. Please put all of your answers in a blue book, or carefully staple your work together in the proper order.

Problem #1 (20 Points): manipulator Jacobian and singularities.

Consider the 3-jointed RPR manipulator in Figure 1. Assume that the first axis is vertical. The second (prismatic) axis is orthogonally intersects the first axis. The third axis orthogonally intersects the prismatic joint axis, and is simultaneously orthogonal to the first axis. We are only concerned with position the origin of the end-effector frame.

Compute the spatial Jacobian matrix for this manipulator.

Describe its singular configurations.

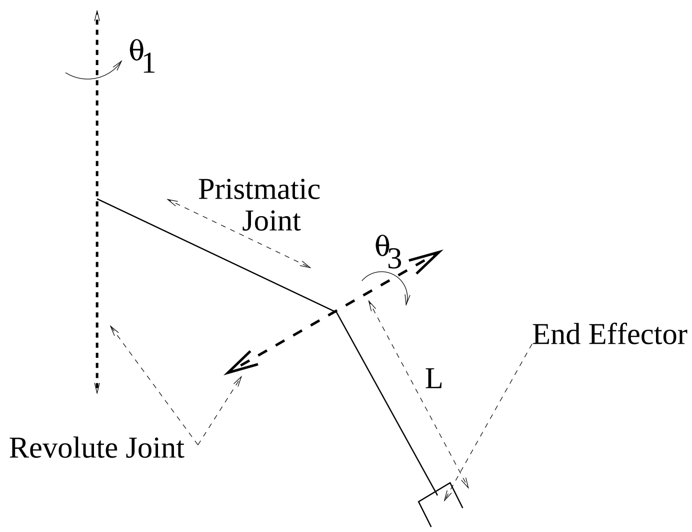

Figure 1: RPR manipulator

Problem #2 (20 Points): Special Configurations of "Slider-Crank" linkages.

Consider the "slider-crank" four bar linkage shown Fig. 2. This mechanism, which is commonly used as the piston mechanism in an internal combustion engine, consists of three revolute joints and one prismatic joint. The joints are numbered successively, with the first joint being at the left of the figure, and the fourth joint being the prismatic joint.

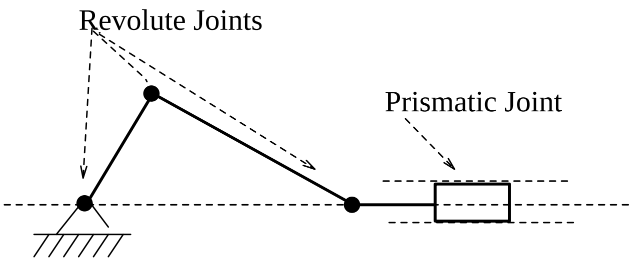

Figure 2: Slider Crank Mechanism

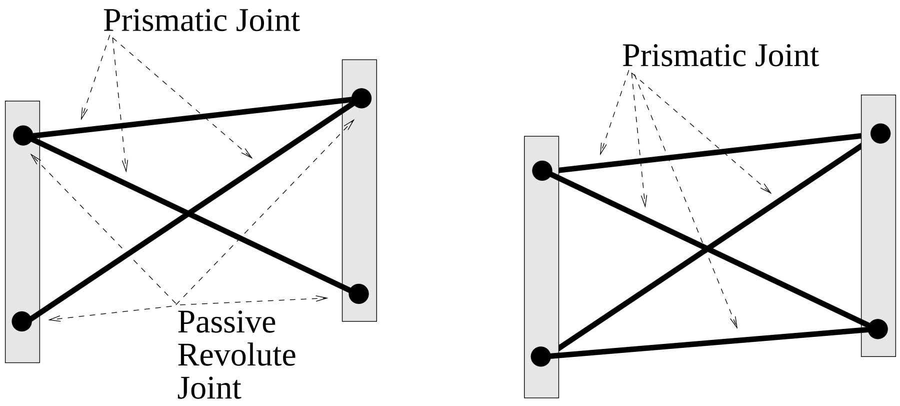

Figure 3: (a) planar parallel mechanism with 3 active prismatic joints; (b) planar parallel mechanism with 4 active prismatic joints.

Part (a): One of the special configurations is obvious: the piston (or prismatic joint) is at "top dead center," and the cylinder comes momentarily to rest in this position. Show that this configuration is indeed a special configuration. In particular, show that joint 4 has a stationary configuration when $\theta_1 = 0$ and $\theta_2 = 0$.

Part (b): Develop an expression for the stationary configurations of joint 1. What are the necessary conditions for joint 1 to have a stationary configuration?

Problem #3 (20 Points): parallel mechanisms

Figure 3(a) shows a planar "crossed" parallel mechanism. It consists of three active prismatic joints and six passive revolute joints.

Part (a): Write the structure equations for this system.

Part (b): Find and sketch the kinematic and actuator singular configurations of this mechanism.

Figure 3(b) is similar to Figure 3(a), except that it has an extra (or fourth) prismatic joint. Such a mechanism is considered to be "overconstrained."

Part (c): Can you describe some possible advantages and disadvantages that arise from overconstraint. What are the singular configurations of this mechanism?

Problem #4 (20 Points): differential geometry of curves

Let $\alpha\colon I \to \mathbb{R}^2$ be a regular parametrized plane curve. The curve:

$$\beta(t) = \alpha(t) + \frac{1}{\kappa(t)}\vec{n}(t)$$

is known as the *evolute* of $\alpha(t)$. $\kappa(t)$ is the curvature at t, while $\vec{n}$ is the unit normal vector at t. Evolutes are important in gear theory.

(a) Show that the tangent at t of the evolute of $\alpha(t)$ is the normal to α at t.

(b) what is the evolute of a circle?

Problem #5 (20 Points): (force closure)

An equilateral triangle is grasped by three point contact fingers (Figure 4). Each finger contact lies at the midpoint of each triangle edge. Assume that finger contacts #1 and #2 are frictionless. Assume that there is friction at contact #3.

Part (a): Sketch the structure of the grasp map for this grasp

Part (b): Is this grasp force closure? Justify your answer using one of the force closure definitions.

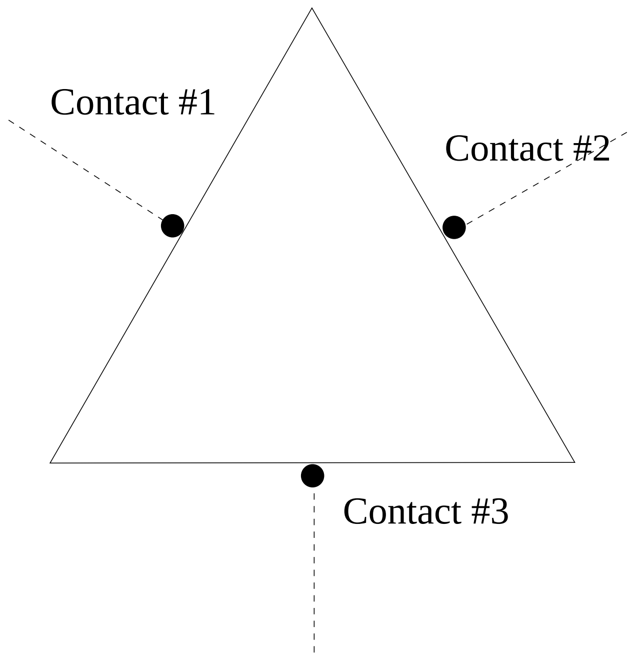

Figure 4: Planar Object