ME 115(b): Final Exam, Spring 2000/2001

Instructions

1. Limit your total time to 5 hours. That is, it is okay to take a break in the middle of the exam if you need to ask me a question, or go to dinner, etc.
2. You may use any class notes, books, or other written material.
3. You may use mathematica or any software or computational tools to assist you.
4. Feel free to ask me or the T.A.s questions about the exam.
5. The final is due by 5:00 p.m. on the last day of the final period.
6. The point values are listed for each problem to assist you in allocation of your time.
7. Please put all of your answers in a blue book, or carefully staple your work together in the proper order.

Problem #1: (25 Points)

Consider the planar mechanism shown in Figure 1. This is a "geared" 6-bar mechanism which I have recently used as a leg thrusting mechanism for a "hopping" robot. Note that the spring is inconsequential for your analysis. The gears in this mechanism have the same radius. That is, as one gear in a gear pair turns, the other gear rotates at the same angular velocity, but opposite sense of direction.

1. What is the mobility of this mechanism?
2. Sketch the form of the structure equations.
3. Sketch the velocity equations.
4. Physically describe the conditions under which this mechanism becomes singular, and interpret it in light of the above derivations.

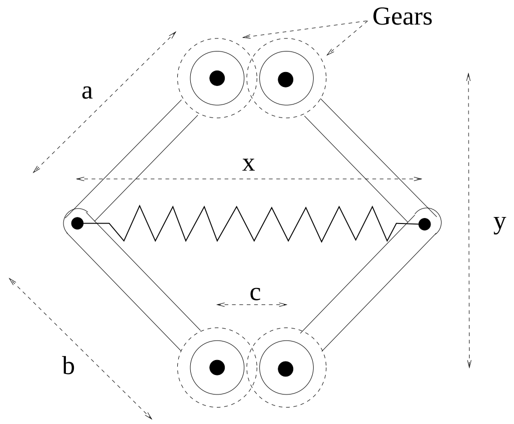

Figure 1: Planar 6-bar Geared Mechanism

Problem #2: (20 Points)

Consider the 3-jointed "cylindrical" robot manipulator shown in Figure 2.

1. Determine the spatial Jacobian Matrix for this mechanism.
2. Determine and physically interpret the singularities of the mechanism.

Problem #3 (20 Points)

In class we discussed a variety of "redundancy resolution" techniques. One method which we didn't discuss is the "Augmented Jacobian" technique. In this method, one "augments" the "task vector" by identifying additional "tasks" so that the resulting systems is no longer

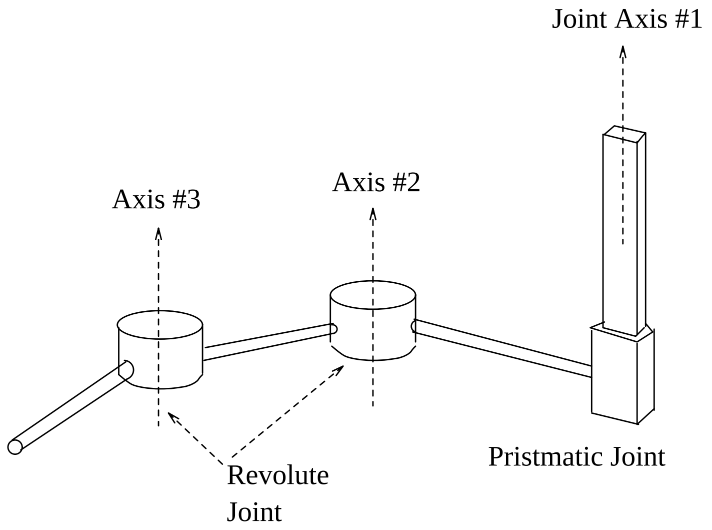

Figure 2: Schematic of a "Cylindrical Manipulator"

redundant. For example, let $\vec{x}$ be the location of the end-effector. $\vec{x}$ is related to the manipulator joint angles through the forward kinematics relationship:

$$\vec{x} = f(\vec{\theta})$$

where $\vec{\theta}$ is an n-vector of joint angles and $\vec{x}$ represents the p independent end-effector coordinates. One can define $(n-p)$ additional "tasks":

$$\vec{\phi} = g(\vec{\theta})$$

where $\vec{\phi}$ is the $(n-p)\times 1$ "augmented task vector." For example, one might define elements of ϕ as the orientation or elevation of interior links. Using the augmented task vector, one can then define the "Augmented Jacobian" through the relationship:

$$\begin{bmatrix} \dot{\vec{x}} \\ \dot{\vec{\phi}} \end{bmatrix} = \begin{bmatrix} \mathbf{J} \\ \frac{\partial g(\vec{\theta})}{\partial \vec{\theta}} \end{bmatrix}$$

(a) Describe the conditions under which the Augmented Jacobian loses rank.

(b) Physically interpret each of the conditions in Part (a).

Problem #4 (15 points): Derive the contact equations for one planar ellipse rolling/sliding on another planar ellipse. Recall that the boundary of the ellipse can be parametrized as:

$$\begin{bmatrix} x(u) \\ y(u) \end{bmatrix} = \begin{bmatrix} a\cos(u) \\ b\sin(u) \end{bmatrix}$$

Note that when $a = b$, the ellipse is a circle. Derived the equations for the condition shown in Figure 3 Under what conditions does the relative curvature become ill defined?

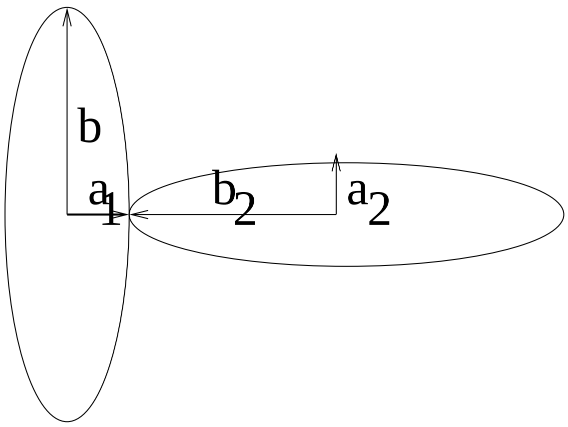

Figure 3: Rolling Ellipses

Problem #5 (20 Points): (force closure)

A 3-dimensional body (such as an ellipsoid) is grasped by two fingers in an antipodal point grasp (Figure 4). Let one of the contacts be modelled by the point contact with friction model. Let the other contact be modelled by a soft finger contact.

Part (a): Sketch the structure of the grasp map for this grasp

Part (b): Is this grasp force closure? Justify your answer using one of the force closure definitions.

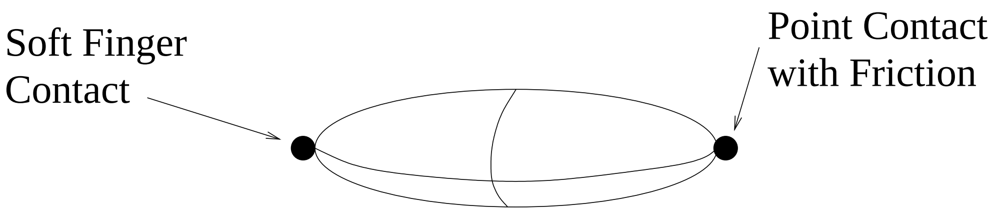

Figure 4: 2-fingered grasp of 3-dimensional Object