ME 115(a): Final Exam
(Winter Quarter 2003/2004)

Instructions

1. Limit your total time to 5 hours. That is, it is okay to take a break in the middle of the exam if you need to ask a question, or go to dinner, etc.
2. You may use any class notes, books, or other written material. You may not discuss this final with other class students or other people except me or the T.A..
3. You may use mathematica or any software or computational tools to assist you. However, if you find that your solution approach requires a lot of algebra or a lot of computation, then you are probably taking a less than optimal approach.
4. The final is due by 5:00 p.m. on the last day of finals.
5. The point values are listed for each problem to assist you in allocation of your time.

Problem 1: (35 Points)

The first three joints of the "armatron" manipulator that we used occasionally for demonstrations in class looks like Figure 1

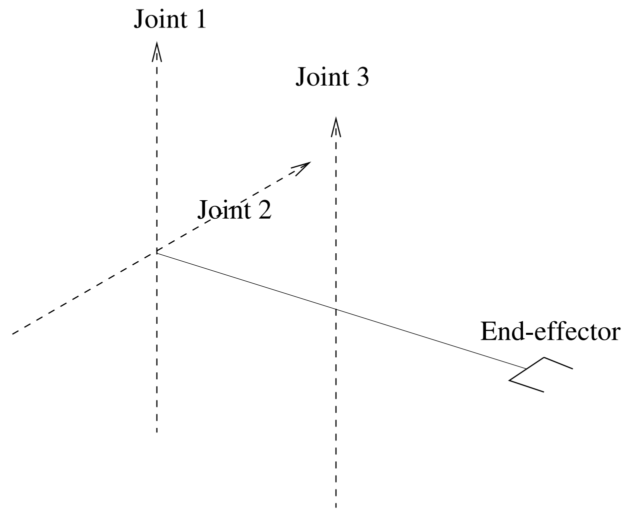

Figure 1: Schematic of Armatron Manipulator Geometry

Part (a): (5 points) Determine the Denavit-Hartenberg parameters.

Part (b): (7 points) Using either the Denavit-hartenberg approach or the product-of-exponentials approach, determine the forward kinematics.

Part (c): (8 points) Develop an expression for the spatial Jacobian matrix for this manipulator.

Part (d): (15 points) Solve the inverse kinematics of this manipulator, assuming that the goal is to position the origin of the tool frame.

Problem 2: (15 Points)

We discovered numerous ways to represent and manipulate spatial displacements. Those crazy kinematicians have yet another variation on the same theme using something called "dual numbers." A dual number, $\tilde{a}$, takes the form:

$$\tilde{a} = a_r + \epsilon \; a_d$$

where a_r is the "real" part of the dual number and a_d is the "dual" or "pure" part of the dual number. The bases for the dual numbers are 1 and ϵ, and they obey the rules:

$$\begin{aligned} 1 \cdot 1 &= 1 \\ 1 \cdot \epsilon &= \epsilon \cdot 1 = \epsilon \\ \epsilon^2 &= 0 \end{aligned}$$

Dual numbers have many interesting properties, though we will only explore one aspect of their characteristics in this problem.

Part (a): (10 points). We can represent spatial displacments as "dual rotation matrices." That is, if a spatial displacement has the form:

$$g = \begin{bmatrix} R & \overline{p} \\ \overline{0}^T & 1 \end{bmatrix}$$

where $R \in SO(3)$ and $\overline{p} \in \mathbb{R}^3$, then the dual representation of the spatial displacement is:

$$\tilde{g} = R + \epsilon(\hat{p}R)$$

1. Show that $\tilde{g}$ is an orthogonal matrix.
2. If g_1 and g_2 are spatial displacements, and $\tilde{g}_1$ and $\tilde{g}_2$ there dual equivalents, then show that $g_1\ g_2$ and $\tilde{g}_1\ \tilde{g}_2$ are equivalent.

Hint: in some ways of solving this problem, it might be useful to recall that if $A \in SO(3)$ and $\overline{v} \in \mathbb{R}^3$, then $\widehat{(A\overline{v})} = A\hat{v}A^T$).

Part (b): (5 Points). We can also use dual numbers to represent twist coordinates. Let $\xi = [\overline{V}, \overline{\omega}]^T$ be a vector twist coordinates. Its dual representation is $\tilde{\xi} = \overline{\omega} + \epsilon\overline{V}$. Show that

1. if g is a spatial displacement, and ξ is a twist, then $Ad_g\xi$ is equivalent to $\tilde{g}\tilde{\xi}$.
2. If ξ_1 and ξ_2 are two twists, then the dual part of dual dot product $\tilde{\xi}_1 \cdot \tilde{\xi}_2$ is equivalent to the reciprocal product of ξ_1 and ξ_2. (Note, the real part of this product is called the "Klein product.").

Problem 3: (20 Points)

Next quarter we will extensively study the problem of grasping–i.e., how one can grab an object with fingers in such as way as to prevent the grasped object from slipping out of the grasp. Consider a planar disc which is touched by 3 "planar" fingers (Figure 2. Assume that each finger touches the disc with *frictionless* point contact. Also assume that each finger can apply any possible force to the object that will be supported by a frictionless contact.

Question: Is the disc immobilized? That is, are there any free motions of the disc that can not be prevented by the fingers? In addition to an intuitive discussion of this question, you must back up your answer with some analysis.

Problem 4: (Mobility, 20 points total)

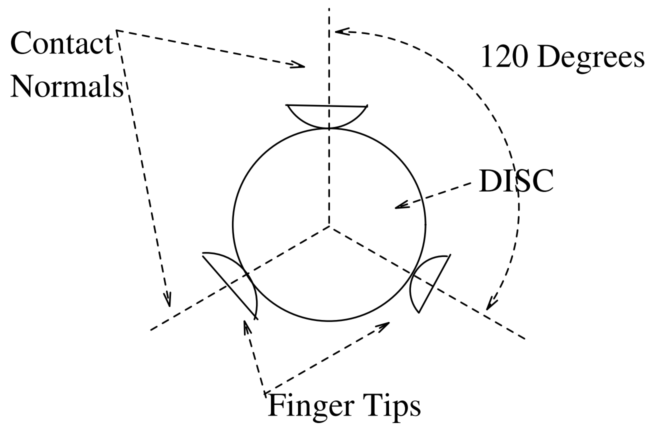

Figure 2: 3-fingered grasp of a disc

Part (a): (10 Points) The left side of Figure 3 shows a *geared 6-bar linkage*. This mechanism consists of 6 links connected by 6 revolute joints. Additionally, there are two pairs of gears included in the mechanism, as shown in the figure. These gears are connected to the underlying joints/links, so that the joints associated with the gear pair rotate at the same speed, but with opposite sign. What is the mobility of this mechanism? While an intuitive discussion is useful, please try to back up your result with some analysis.

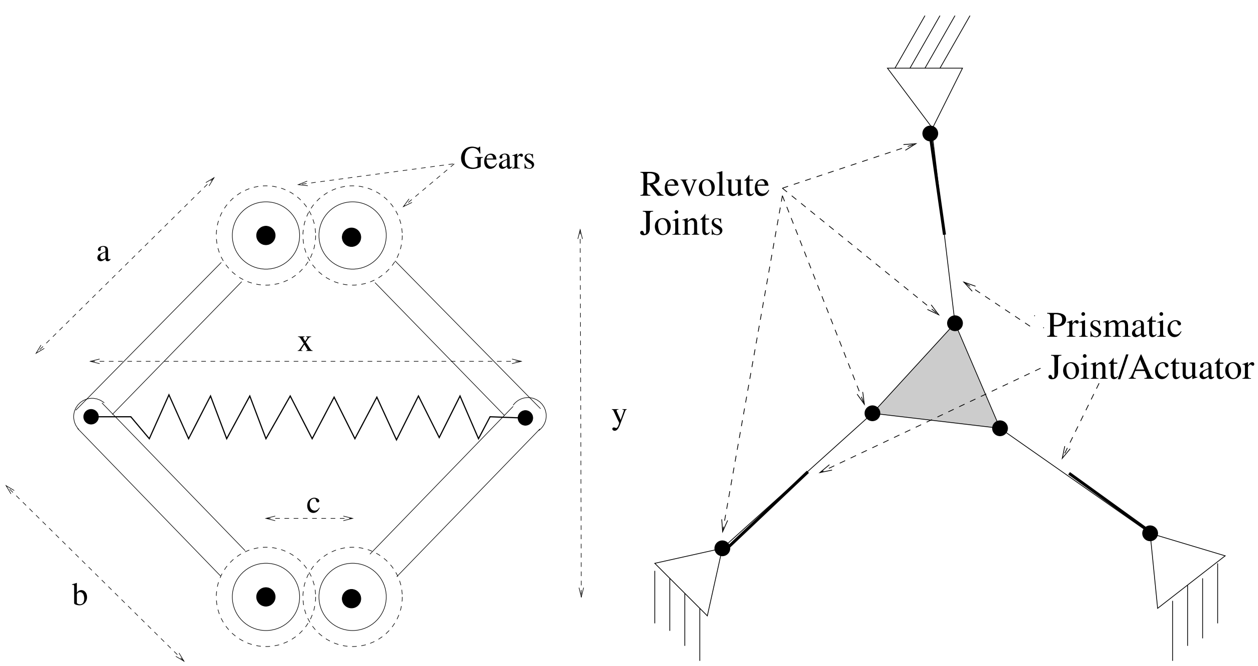

Figure 3: **Left Figure:** Schematic diagram of geared 6-bar linkage. Note that the jagged line across the middle represents a spring. **Right Figure:** Planar mechanism consisting of revolute and prismatic joints.

Part (b): (10 Points) The right side of Figure 3 shows a planar "parallel" mechanism comprised of rigid body links, prismatic joints, and revolute joints as shown in the figure. What is the mobility of this mechanism? While we have not yet discussed parallel mechanisms in class, the meaning of mobility is the same in this case. THat is, how many independent degrees of freedom are necessary to uniquely specify the state of this mechanism?

Problem 5: (10 points)

Let g be a homogeneous transformation matrix representing the displacement of a planar rigid body:

$$g = \begin{bmatrix} R & \bar{d} \\ \bar{0}^T & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & d_x \\ \sin\theta & \cos\theta & d_y \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

where R and d respectively represent the rotation (by angle θ) and translation of the moving reference frame due to the displacement.

a. Show that the pole of the displacement (in homogeneous coordinates) is an eigenvector of g with eigenvalue 1.

b. (5 extra credit) describe the other two eigenvectors of g.