ME 115(a): Final Exam
(Winter Quarter 2005/2006)

Instructions

1. Limit your total time to 5 hours. That is, it is okay to take a break in the middle of the exam if you need to ask a question, or go to dinner, etc.
2. You may use any class notes, books, or other written material. You may not discuss this final with other class students or other people except me or the T.A..
3. You may use mathematica or any software or computational tools to assist you. However, if you find that your solution approach requires a lot of algebra or a lot of computation, then you are probably taking a less than optimal approach.
4. The final is due by 5:00 p.m. on the last day of finals.
5. The point values are listed for each problem to assist you in allocation of your time.

Problem 1: (15 points)

Consider the planar object shown in Figure 1 which is grasped by two frictionless fingers. Assume that the contact normals of both finger contacts are collinear. Describe the set of possible planar motions of the object that can not be prevented by any action of the fingers. Use some analysis to back up your discussion.

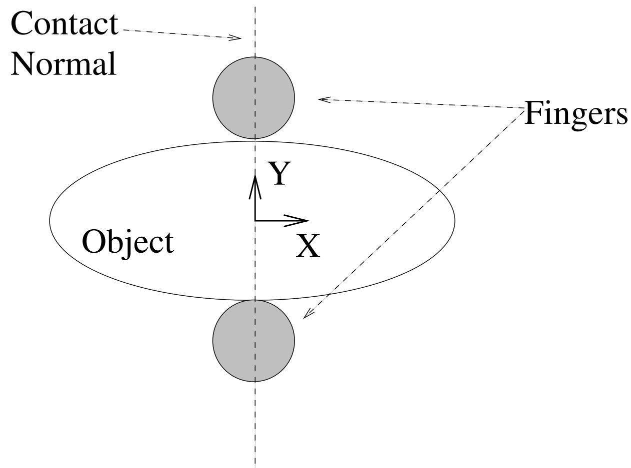

Figure 1: Schematic of two-fingered frictionless grasp

Problem 2: (15 points)

For the rotation matrix given below

$$R = \begin{bmatrix} 0.833333 & -0.186887 & 0.52022 \\ 0.52022 & 0.583333 & -0.623773 \\ -0.186887 & 0.79044 & 0.583333 \end{bmatrix}$$

Part (a): (7 points) Compute the axis of rotation and angle of rotation.

Part (b): (4 points) Determine the unit quaternion that is equivalent to this rotation.

Part (c): (4 points) What are the z-y-z Euler angles of this rotation?

Problem 3: (20 points)

We discovered numerous ways to represent and manipulate spatial displacements. Those crazy kinematicians have yet another variation on the same theme using something called "dual numbers." A dual number, $\tilde{a}$, takes the form:

$$\tilde{a} = a_r + \epsilon\ a_d$$

where a_r is the "real" part of the dual number and a_d is the "dual" or "pure" part of the dual number. The bases for the dual numbers are 1 and ϵ, and they obey the rules:

$$\begin{aligned} 1 \cdot 1 &= 1 \\ 1 \cdot \epsilon &= \epsilon \cdot 1 = \epsilon \\ \epsilon^2 &= 0 \end{aligned}$$

Dual numbers have many interesting properties, though we will only explore one aspect of their characteristics in this problem.

Part (a): (15 points). We can represent spatial displacments as "dual rotation matrices." That is, if a spatial displacement has the form:

$$g = \begin{bmatrix} R & \overline{p} \\ \overline{0}^T & 1 \end{bmatrix}$$

where $R \in SO(3)$ and $\overline{p} \in \mathbb{R}^3$, then the dual representation of the spatial displacement is:

$$\tilde{g} = R + \epsilon(\hat{p}R)$$

1. Show that $\tilde{g}$ is an orthogonal matrix.
2. If g_1 and g_2 are spatial displacements, and $\tilde{g}_1$ and $\tilde{g}_2$ there dual equivalents, then show that $g_1\ g_2$ and $\tilde{g}_1\ \tilde{g}_2$ are equivalent.

Hint: in some ways of solving this problem, it might be useful to recall that if $A \in SO(3)$ and $\overline{v} \in \mathbb{R}^3$, then $\widehat{(A\overline{v})} = A\hat{v}A^T$).

Part (b): (5 Points). We can also use dual numbers to represent twist coordinates. Let $\xi = [\overline{V}, \overline{\omega}]^T$ be a vector twist coordinates. Its dual representation is $\tilde{\xi} = \overline{\omega} + \epsilon\overline{V}$. Show that

1. if g is a spatial displacement, and ξ is a twist, then $Ad_g\xi$ is equivalent to $\tilde{g}\tilde{\xi}$.
2. If ξ_1 and ξ_2 are two twists, then the dual part of dual dot product $\tilde{\xi}_1 \cdot \tilde{\xi}_2$ is equivalent to the reciprocal product of ξ_1 and ξ_2. (Note, the real part of this product is called the "Klein product.").

Problem 4: (15 Points) Consider the two screws, S_1 and S_2, shown in Figure 2. S_1 is perpendicular to the plane, P, and has zero pitch: $h_1 = 0$. The screw axis of S_2 lies in P, and S_2 some non-zero pitch, h_2. The distance between S_1 and S_2, as measured along a mutually perpendicular line, is denoted a. Describe the set of all screws whose axes lie in P and that are reciprocal to both S_1 and S_2.

Problem 5: (25 Points)

Figure 3 shows a schematic of an 3-jointed PRR robot manipulator. This manipulator consists of one prismatic joint (the first joint) and two revolute joints. All three joint axes are vertical.

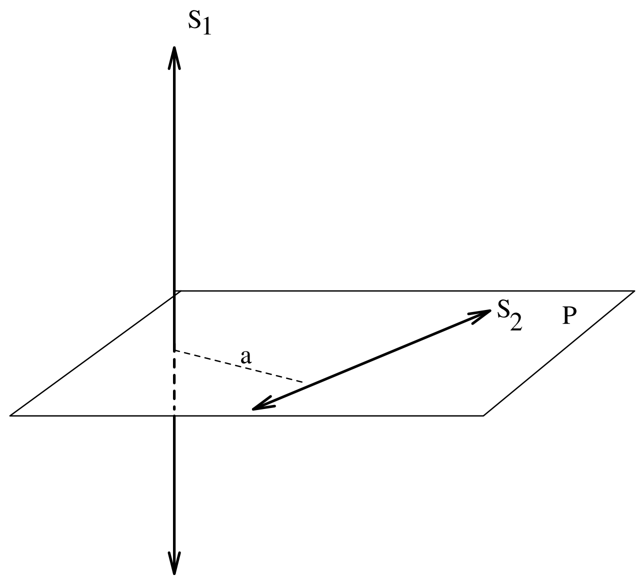

Figure 2: Two Screws

(a) (5 points) Derive the Denavit-Hartenberg parameters.

(b) (5 points) Derive the forward kinematic equations of this robot.

(c) (5 points) Derive the spatial Jacobian matrix for this system.

(d) (10 points) Derive the inverse kinematics for this robot (where you want to position just the tool frame origin at a given spatial position).

Problem 6: (10 points)

Let g be a homogeneous transformation matrix representing the displacement of a planar rigid body:

$$g = \begin{bmatrix} R & \overline{d} \\ \overline{0}^T & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & d_x \\ \sin\theta & \cos\theta & d_y \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

where R and d respectively represent the rotation (by angle θ) and translation of the moving reference frame due to the displacement.

a. Show that the pole of the displacement (in homogeneous coordinates) is an eigenvector of g with eigenvalue 1.

b. (5 extra credit) describe the other two eigenvectors of g.

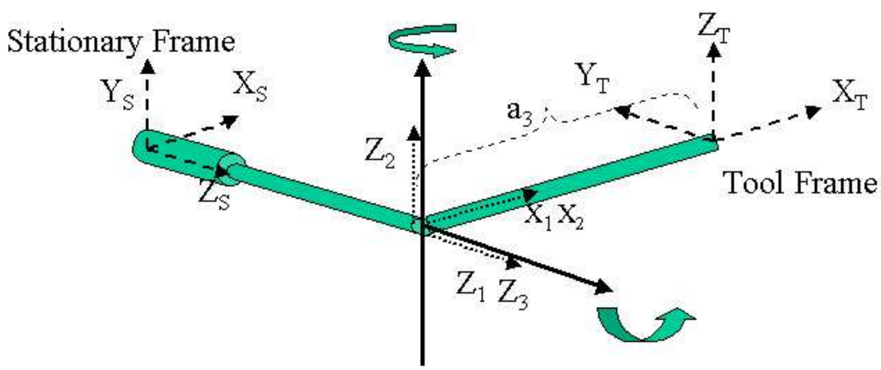

Figure 3: Schematic of a PRR Manipulator"