ME 115(a): Final Exam
(Winter Quarter 2007/2008)

Instructions

1. Limit your total time to 5 hours. That is, it is okay to take a break in the middle of the exam if you need to ask a question, or go to dinner, etc.
2. You may use any class notes, books, or other written material. You may not discuss this final with other class students or other people except me or the T.A..
3. You may use mathematica or any software or computational tools to assist you. However, if you find that your solution approach requires a lot of algebra or a lot of computation, then you are probably taking a less than optimal approach.
4. The final is due by 5:00 p.m. on the last day of finals.
5. The point values are listed for each problem to assist you in allocation of your time.

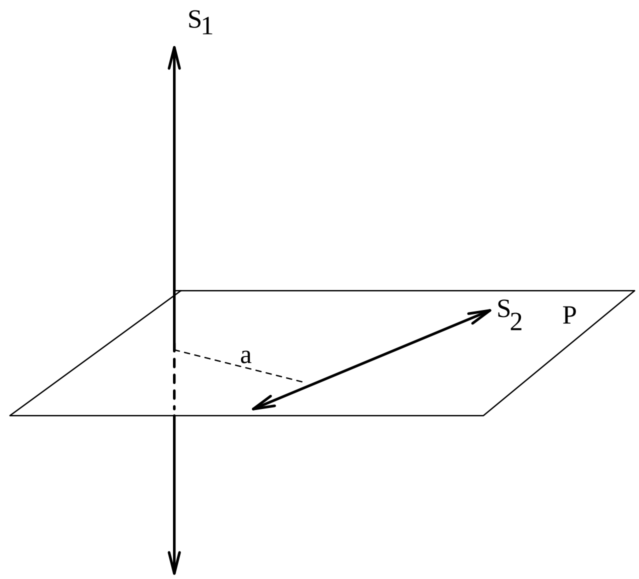

Figure 1: Two Screws

Problem 1: (15 points)

Consider the two screws, S_1 and S_2, shown in Figure 1. S_1 is perpendicular to the plane, P, and has zero pitch: $h_1 = 0$. The screw axis of S_2 lies in P, and S_2 some non-zero pitch, h_2. The distance between S_1 and S_2, as measured along a mutually perpendicular line, is denoted a. Describe the set of all screws whose axes lie in P and that are reciprocal to both S_1 and S_2.

Problem 2: (20 points)

We discovered numerous ways to represent and manipulate spatial displacements. Those crazy kinematicians have yet another variation on the same theme using something called "dual numbers." A dual number, $\tilde{a}$, takes the form:

$$\tilde{a} = a_r + \epsilon\ a_d$$

where a_r is the "real" part of the dual number and a_d is the "dual" or "pure" part of the dual number. The bases for the dual numbers are 1 and ϵ, and they obey the rules:

$$\begin{aligned} 1 \cdot 1 &= 1 \\ 1 \cdot \epsilon &= \epsilon \cdot 1 = \epsilon \\ \epsilon^2 &= 0 \end{aligned}$$

Dual numbers have many interesting properties, though we will only explore one aspect of their characteristics in this problem.

Part (a): (15 points). We can represent spatial displacments as "dual rotation matrices." That is, if a spatial displacement has the form:

$$g = \begin{bmatrix} R & \overline{p} \\ \overline{0}^T & 1 \end{bmatrix}$$

where $R \in SO(3)$ and $\overline{p} \in \mathbb{R}^3$, then the dual representation of the spatial displacement is:

$$\tilde{g} = R + \epsilon(\hat{p}R)$$

1. Show that $\tilde{g}$ is an orthogonal matrix.
2. If g_1 and g_2 are spatial displacements, and $\tilde{g}_1$ and $\tilde{g}_2$ there dual equivalents, then show that $g_1\ g_2$ and $\tilde{g}_1\ \tilde{g}_2$ are equivalent.

Hint: in some ways of solving this problem, it might be useful to recall that if $A \in SO(3)$ and $\overline{v} \in \mathbb{R}^3$, then $\widehat{(A\overline{v})} = A\hat{v}A^T$.

Part (b): (5 Points). We can also use dual numbers to represent twist coordinates. Let $\xi = [\overline{V}, \overline{\omega}]^T$ be a vector of twist coordinates. Its dual representation is $\tilde{\xi} = \overline{\omega} + \epsilon\overline{V}$. Show that

1. if g is a spatial displacement, and ξ is a twist, then $Ad_g\xi$ is equivalent to $\tilde{g}\tilde{\xi}$.
2. If ξ_1 and ξ_2 are two twists, then the dual part of dual dot product $\tilde{\xi}_1 \cdot \tilde{\xi}_2$ is equivalent to the reciprocal product of ξ_1 and ξ_2. (Note, the real part of this product is called the "Klein product.").

Problem 3: (10 Points)

Consider the three jointed manipulator shown in Figure 2, which you already considered in a homework problem. This first two joints of this manipulator are revolute joints, while the third joint is a prismatic joint. The joint axes are parallel to each other.

Part (a): Find the hybrid Jacobian matrix of this manipulator.

Problem 4: (10 Points) Find the singularities of the manipulator analyzed in Problem 3.

Problem 5: (25 Points) The first three joints of the "armatron" manipulator (a toy sold by Radio Shack!) are shown in Figure 3

Part (a): (3 points) Determine the Denavit-Hartenberg parameters.

Part (b): (7 points) Using either the Denavit-hartenberg approach or the product-of-exponentials approach, determine the forward kinematics. That is, relate the coordinates of the origin of the tool frame to the joint angles.

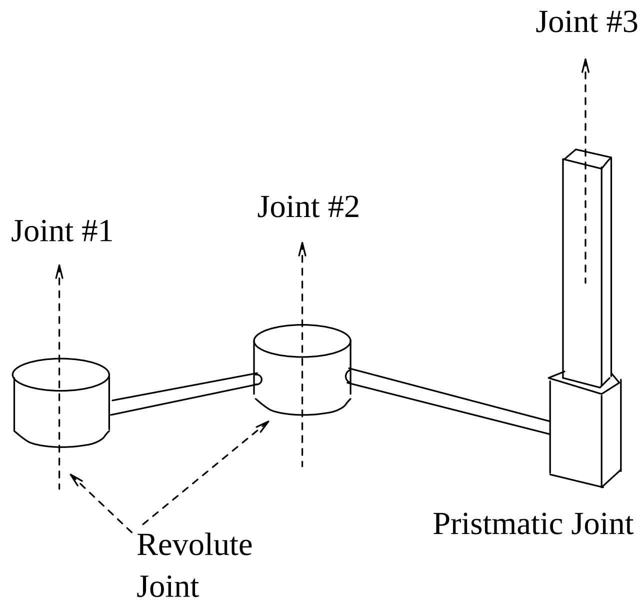

Figure 2: Schematic of RRP Manipulator Geometry

Part (c): (15 points) Solve the inverse kinematics of this manipulator, assuming that the goal is to position the origin of the tool frame. You can use a geometric, algebraic, Paden-Kahan, or other approach.

Problem 6: (10 points)

Let g be a homogeneous transformation matrix representing the displacement of a planar rigid body:

$$g = \begin{bmatrix} R & \overline{d} \\ \overline{0}^T & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & d_x \\ \sin\theta & \cos\theta & d_y \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

where R and $\overline{d} = [d_x \ d_y]^T$ respectively represent the rotation (by angle θ) and translation of the moving reference frame due to the displacement.

a. Show that the pole of the displacement (in homogeneous coordinates) is an eigenvector of g with eigenvalue 1.

b. (5 extra credit) describe the other two eigenvectors of g.

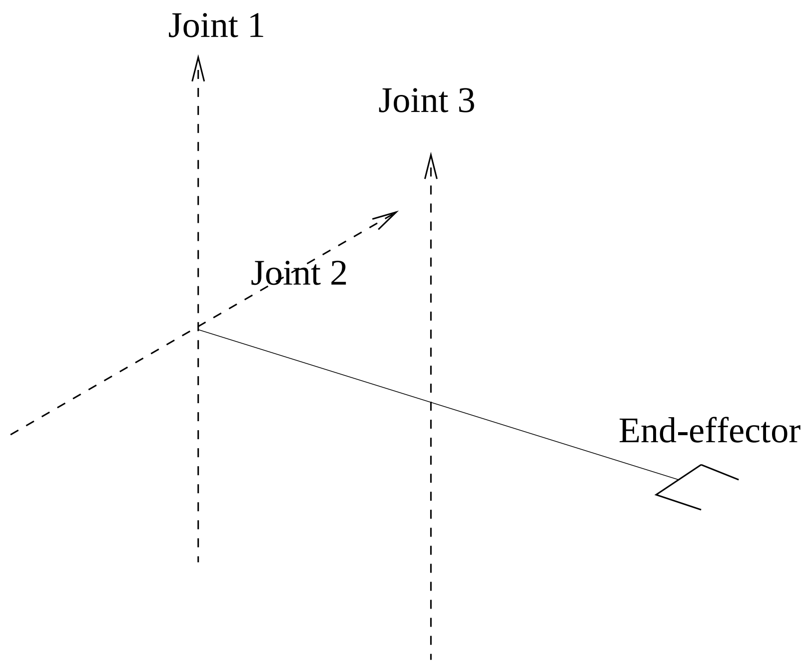

Figure 3: Schematic of Armatron Manipulator Geometry