ME 115(a): Final Exam
(Winter Quarter 2013/2014)

Instructions

1. Limit your total time to 5 hours. That is, it is okay to take a break in the middle of the exam if you need to ask a question, or go to dinner, etc.
2. You may use any class notes, books, or other written material. You may not discuss this final with other class students or other people except me or the class Teaching Assistants.
3. You may use Mathematica, MATLAB, or any software or computational tools to assist you. However, if you find that your solution approach requires a lot of algebra or a lot of computation, then you are probably taking a less than optimal approach.
4. You can not use the internet to solve these problems, except for material on the course web site.
5. The final is due by 5:00 p.m. on the last day of finals.
6. The point values are listed for each problem to assist you in allocation of your time.
7. Please put all of your work in a blue book, or carefully staple the pages of your solution in the proper order.

Problem 1: (35 points) The first three joints of the "armatron" robot manipulator (which was one of Radio Shack's most popular toys over a space of 20 years!) are shown schematically in Figure 1. Note that all three joints are revolute joints.

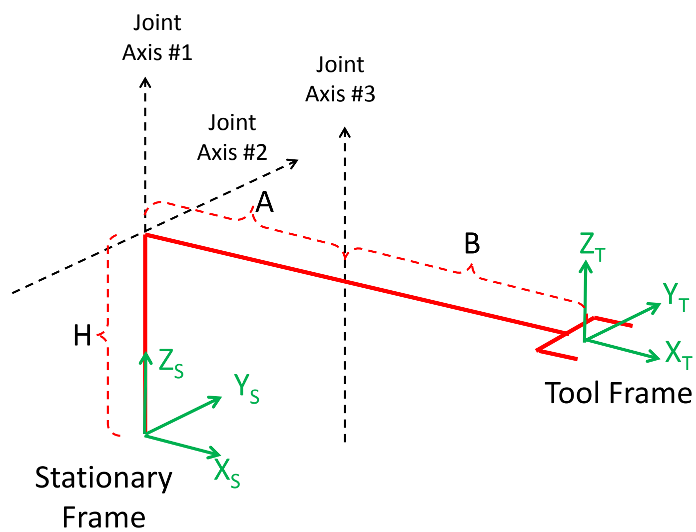

Figure 1: Schematic of Armatron Manipulator Geometry

Part (a): (5 points) Determine the Denavit-Hartenberg parameters of this manipulator.

Part (b): (7 points) Using either the Denavit-Hartenberg or the Product of Exponentials approach, determine the forward kinematics of this manipulator (i.e., determine the transformation between the tool and stationary frames).

Part (c): (8 points) Develop an expression for the spatial Jacobian matrix for this manipulator.

Part (d): (15 points) Develop expressions for the inverse kinematics solution for this manipulator. The goal is to position the origin of the tool frame relative to the origin of the stationary frame.

Problem 2: (10 points)

Let g be a homogeneous transformation matrix representing the displacement of a planar rigid body:

$$g = \begin{bmatrix} R & \bar{d} \\ \bar{0}^T & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & d_x \\ \sin\theta & \cos\theta & d_y \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

where R and d respectively represent the rotation (by angle θ) and translation of the moving reference frame due to the displacement.

a. Show that the pole of the displacement (in homogeneous coordinates) is an eigenvector of g with eigenvalue 1.

b. (5 extra credit) describe the other two eigenvectors of g.

Problem 3: (15 points)

Planar displacements can be represented as a combination of a translation by vector $\overline{d} = [d_x\ d_y]^T$ and a rotation by angle θ, which could be also be represented by a 3×3 homogeneous matrix of the form:

$$g = \begin{bmatrix} \cos\theta & -\sin\theta & d_x \\ \sin\theta & \cos\theta & d_y \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

Or, we also noted that every planar displacement was equivalent to a rotation about a "pole."

Part (a): (7 points) Let a body-fixed reference frame attached to a rigid body be initially in coincidence with the origin of a fixed reference observing frame. Let this body undergo planar displacement by rotation of angle ϕ about a pole located at a distance $\overline{p} = [p_x\ p_y^T$ from the origin of the reference frame. Compute the 3×3 homogeneous transformation matrix that describes this displacment (in terms of ϕ, p_x, and p_y).

We can also represent planar displacements using a special type of quaternion algebra knowns as "planar quaternions." The planar quaternion algebra has basis elements (1, $i\epsilon$, $j\epsilon$, k) where:

- The product of basis elements i, j, k behave just like the quaternion basis elements.
- $\epsilon^2 = 0$.
- i, j, and k, commute with ϵ. E.g., $i\epsilon\ k = ik\ \epsilon = -j\epsilon$ and $i\epsilon\ j\epsilon = ij\epsilon^2 = 0$.

Recall that in the case of unit quaternions, the quaternion coefficients can be identified with the Euler parameters of a rotation, and thus unit quaternions can be used to represent rotations.

Let a planar quaternion have the form:

$$Z = Z_4 + Z_1 i\epsilon + Z_2 j\epsilon + Z_3 k$$

where Z_1, Z_2, Z_3, $Z_4 \in \mathbb{R}^n$. The coefficients of the planar quaternion can be identified with the planar displacement parameters as follows:

$$\begin{aligned} Z_4 &= \cos(\tfrac{\phi}{2}) \\ Z_3 &= \sin(\tfrac{\phi}{2}) \\ Z_2 &= -p_x \sin(\tfrac{\phi}{2}) \\ Z_1 &= p_y \sin(\tfrac{\phi}{2}) \end{aligned} \tag{3}$$

where $\overline{p} = [p_x \; p_y]^T$ is the pole of the planar displacement.

Part (b): (15 points)

Using your results from part (a), compute ϕ, d_x, and d_y in terms of Z_1, Z_2, Z_3, and Z_4.

Note: these results are useful because of the following facts, *which you need not prove.* Let $\overline{v} = (x, y, 1)$ be a planar vector in homogeneous coordinates. This vector can be associated with the "pure" planar quaternion:

$$v = (yi\epsilon \; - \; xj\epsilon \; + \; k).$$

If Z is a planar quaternion representing a planar displacement with parameters θ, d_x, and d_y, then it can be shown that:

$$\overline{v}' = ZvZ^* = (x\sin\phi + y\cos\phi + d_y)i\epsilon - (x\cos\phi - y\sin\phi + d_x)j\epsilon + k \tag{4}$$

and thus this operation is equivalent to:

$$\overline{v}' = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \overline{v} + \begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

Thus, planar quaternions are yet another means to represent planar displacements and coordinate transformations.

Problem 1: (15 points)

Consider the three screws, S_1, S_2, and S_3, shown in Figure 2. All three screws are perpendicular to a plane, P, and pass through the corners of an equilateral triangle (whose sides have dimension d). Each of the three screws has zero pitch. Describe the set of all screws which are simultaneously reciptrocal all three screws.

Problem 5: (Mobility, 20 points total)

Part (a): (10 Points) The left side of Figure 3 shows a *geared 6-bar linkage.* This mechanism consists of 6 links connected by 6 revolute joints. Additionally, there are two pairs of gears included in the mechanism, as shown in the figure. These gears are connected to the underlying joints/links, so that the joints associated with the gear pair rotate at the same

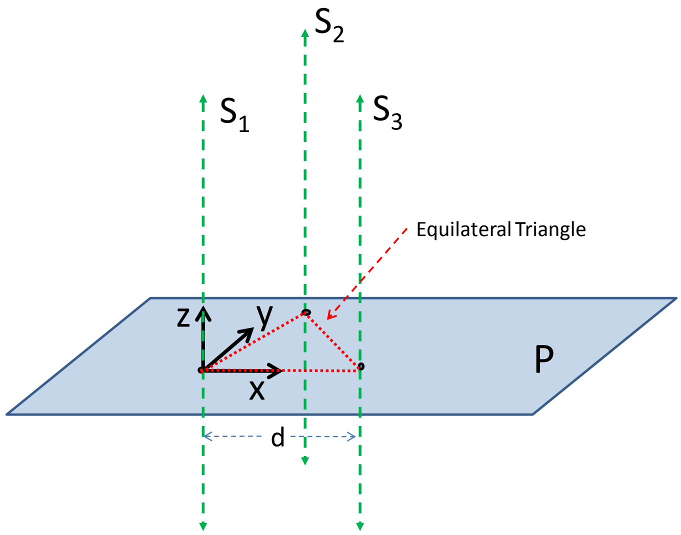

Figure 2: Three Screws

speed, but with opposite sign. What is the mobility of this mechanism? While an intuitive discussion is useful, please try to back up your result with some analysis.

Part (b): (10 Points) The right side of Figure 3 shows a planar "parallel" mechanism comprised of rigid body links, prismatic joints, and revolute joints as shown in the figure. What is the mobility of this mechanism? While we have not yet discussed parallel mechanisms in class, the meaning of mobility is the same in this case. THat is, how many independent degrees of freedom are necessary to uniquely specify the state of this mechanism?

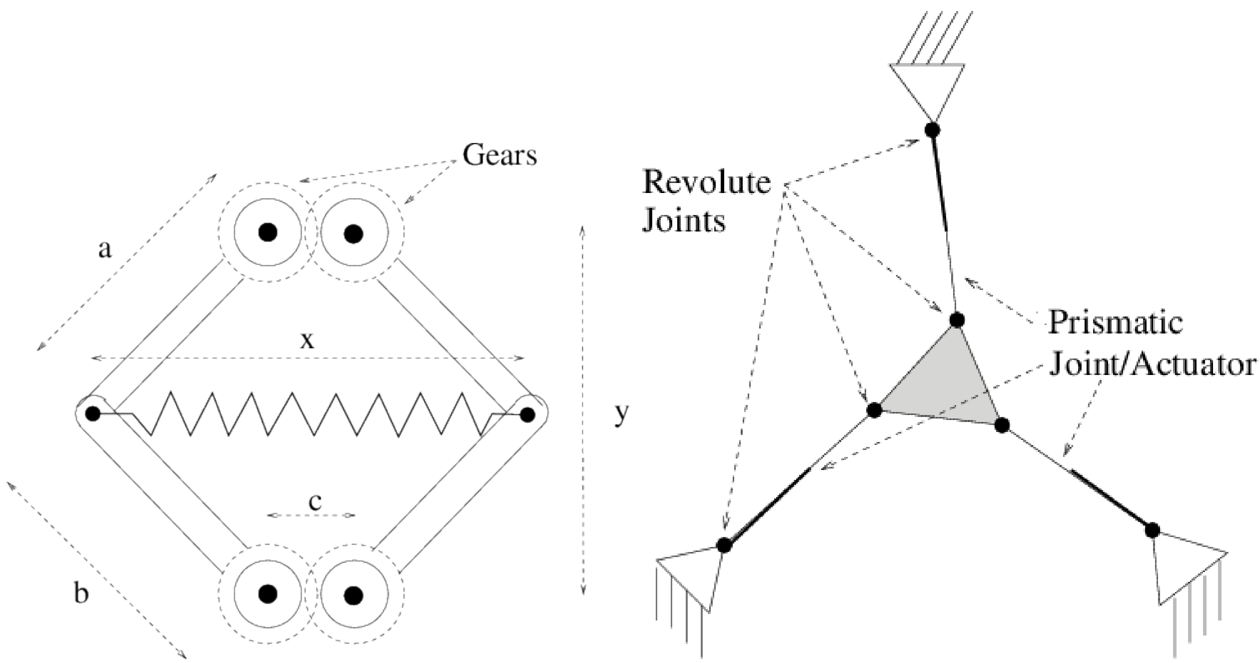

Figure 3: **Left Figure:** Schematic diagram of geared 6-bar linkage. Note that the jagged line across the middle represents a spring. **Right Figure:** Planar mechanism consisting of revolute and prismatic joints.