ME 115(a): Final Exam, 1999/2000

Instructions

1. Limit your total time to 5 hours. That is, it is okay to take a break in the middle of the exam if you need to ask me a question, or go to dinner, etc.
2. You may use any class notes, books, or other written material. You may not discuss this final with other students or other people except me or the T.A.s.
3. You may use mathematica or any software or computational tools to assist you. However, if you find that solution approach requires a lot of algebra or a lot of computation, then you are probably taking a less than optimal approach.
4. The final is due by 5:00 p.m. on the last day of finals.
5. The point values are listed for each problem to assist you in allocation of your time.

Problem 1: (25 Points)

The first three joints of the "armatron" manipulator that we used for demonstrations in class looks like Figure 1

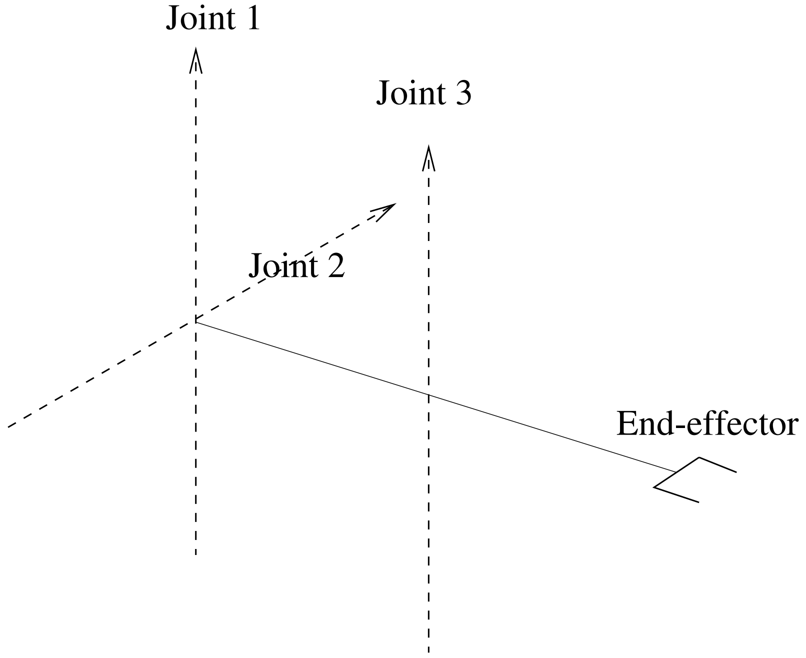

Figure 1: Schematic of Armatron Manipulator Geometry

Part (a): (3 points) Determine the Denavit-Hartenberg parameters.

Part (b): (7 points) Using either the Denavit-hartenberg approach or the product-of-exponentials approach, determine the forward kinematics. That is, relate the coordinates of the origin of the tool frame to the joint angles.

Part (c): (15 points) Solve the inverse kinematics of this manipulator, assuming that the goal is to position the origin of the tool frame.

Problem 2: (25 points)

Let B be a rigid body that is moving about according to $g(t)$, where $g(t) \in SE(3)$ measures the location of a frame fixed to the moving body with respect to a stationary frame. Assume that points P, Q, and R are fixed in this moving body. Further assume that the coordinates of these points, as seen by an observer in a frame affixed to the body, are $\overline{v}_1$, $\overline{v}_2$, $\overline{v}_3$. Assume that at a given instant of time, t_0, we can measure both the location of the points P, Q, and R, as well the velocities of these points, $\dot{P}$, $\dot{Q}$, and $\dot{R}$— as seen by an observer in the stationary frame. Show how to compute either the spatial or the body velocity of B from this information.

The solution to this problem is at the foundation of some "motion capture" algorithms.

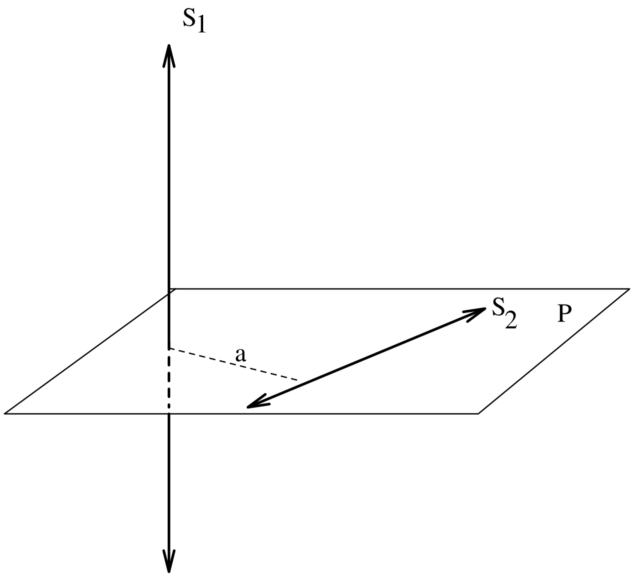

Figure 2: Two Screws

Problem 3: (10 points)

Let g be a homogeneous transformation matrix representing the displacement of a planar rigid body:

$$g = \begin{bmatrix} R & \overline{d} \\ \overline{0}^T & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & d_x \\ \sin\theta & \cos\theta & d_y \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

where R and d respectively represent the rotation (by angle θ) and translation of the moving reference frame due to the displacement.

a. Show that the pole of the displacement (in homogeneous coordinates) is an eigenvector of g with eigenvalue 1.

b. (10 extra credit) describe the other two eigenvectors of g.

Problem 4: (20 Points) Consider the two screws, S_1 and S_2, shown in Figure 2. S_1 is perpendicular to the plane, P, and has zero pitch: $h_1 = 0$. The screw axis of S_2 lies in P, and S_2 some non-zero pitch, h_2. The distance between S_1 and S_2, as measured along a mutually perpendicular line, is denoted a. Describe the set of all screws whose axes lie in P and that are reciprocal to both S_1 and S_2.

Problem 5: (20 Points) Do problem 11(d,e) in Chapter 2 of the MLS text. Recall that you have already done 11(a,b) in Homework #3. You need not rederive any results that were already developed in this homework solution.