ME 115(b): Final Exam/Final Homework

Instructions

1. Limit your total time to 5 hours. That is, it is okay to take a break in the middle of the exam if you need to ask me a question, or go to dinner, etc.
2. You may use any class notes, books, or other written material.
3. You may use mathematica or any software or computational tools to assist you.
4. Feel free to ask me or the T.A.s questions about the exam.
5. The final is due by 5:00 p.m. on Wednesday, March 15.
6. The point values are listed for each problem to assist you in allocation of your time.
7. Please put all of your answers in a blue book, or carefully staple your work together in the proper order.

Problem #1 (30 Points)

In class we discussed a variety of "redundancy resolution" techniques. One method which we didn't discuss is the "Augmented Jacobian" technique. In this method, one "augments" the "task vector" by identifying additional "tasks" so that the resulting systems is no longer redundant. For example, let $\vec{x}$ be the location of the end-effector. $\vec{x}$ is related to the manipulator joint angles through the forward kinematics relationship:

$$\vec{x} = f(\vec{\theta})$$

where $\vec{\theta}$ is an n-vector of joint angles and $\vec{x}$ represents the p independent end-effector coordinates. One can define $(n-p)$ additional "tasks":

$$\vec{\phi} = g(\vec{\theta})$$

where $\vec{\phi}$ is the $(n-p) \times 1$ "augmented task vector." For example, one might define elements of ϕ as the orientation or elevation of interior links. Using the augmented task vector, one can then define the "Augmented Jacobian" through the relationship:

$$\begin{bmatrix} \dot{\vec{x}} \\ \dot{\vec{\phi}} \end{bmatrix} = \begin{bmatrix} \mathbf{J} \\ \frac{\partial g(\vec{\theta})}{\partial \vec{\theta}} \end{bmatrix}$$

(a) Describe the conditions under which the Augmented Jacobian loses rank.

(b) Physically interpret each of the conditions in Part (a).

Problem #2: (30 points)

Consider the planar mechanism shown in Figure 1.

1. What is the mobility of this mechanism?
2. Derive the structure equations.
3. Derive the velocity equations.
4. Physically interpret (using your answer to the last question as a guide if necessary) the conditions under which this mechanism becomes singular.

Problem #3 (15 Points)

Let $\alpha: I \to \mathbb{R}^2$ be a regular parametrized plane curve. The curve:

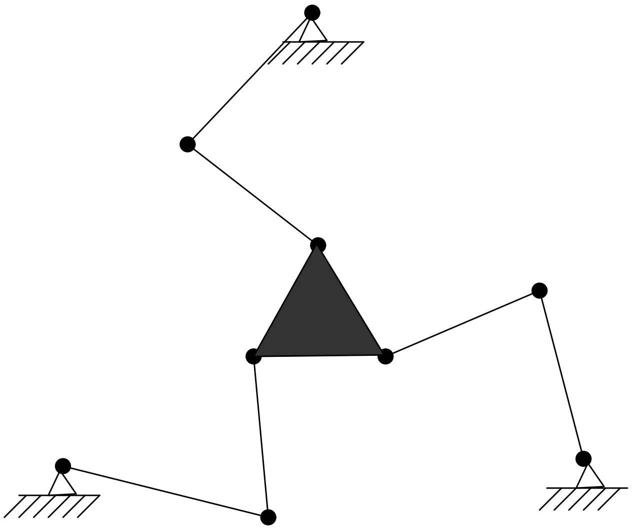

Figure 1: Planar Mechanism

$$\beta(t) = \alpha(t) + \frac{1}{\kappa(t)}\vec{n}(t)$$

is known as the *evolute* of $\alpha(t)$. $\kappa(t)$ is the curvature at t, while $\vec{n}$ is the unit normal vector at t. Evolutes are important in gear theory.

Show that the tangent at t of the evolute of $\alpha(t)$ is the normal to α at t.

Problem #4:

Part (a): Derive the contact equations for an ellipse rolling/sliding on a plane:

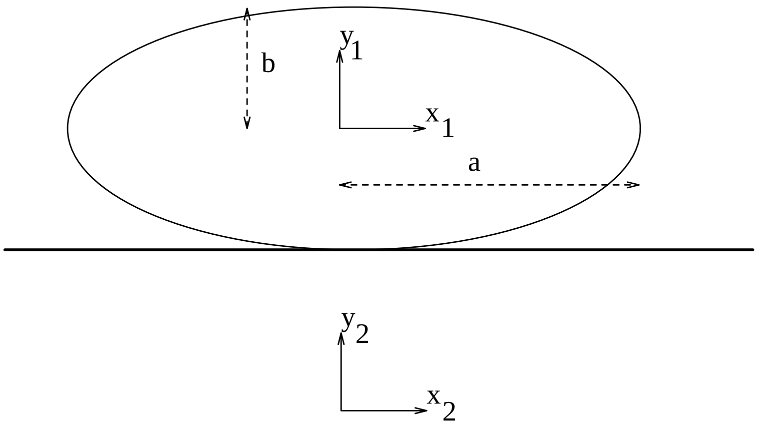

Figure 2: Planar Ellipse

Recall that the boundary of the ellipse can be parametrized as:

$$\begin{bmatrix} x(u) \\ y(u) \end{bmatrix} = \begin{bmatrix} a\cos(u) \\ b\sin(u) \end{bmatrix}$$

Note that when $a = b$, the equations reduce to the case of a disc rolling on a plane.

Part (b): Derive the contact equations for an ellipse rolling/sliding on a circle of radius R. Under what conditions does the relative curvature become ill defined?

Problem #5: (15 Points)

Let the kinematics of a multi-fingered grasp be modeled by the standard model:

$$J_h(\vec{\theta})\dot{\vec{\theta}} = G^T(\vec{\theta})\dot{\vec{x}}$$

where J_h is the $m \times n$ ($n \geq m$) hand Jacobian matrix, G is the $p \times m$ grasp map ($p \leq m$, and p typically is 3 or 6), and $\vec{\theta}$ is the vector of n finger joint angles. m is the sum of of all the dimensions of the finger wrench bases. $\dot{\vec{x}}$ is the object body velocity.

Assume that J_h is surjective and that the grasp is force closure. Show that away from singular configurations, the following equality holds:

$$\begin{aligned} \text{total finger DOF} \; &= \text{object DOF} \\ &+ \text{degrees of internal motion} \\ &+ \text{degrees of internal force} \end{aligned}$$

where "degrees of internal motion" implies the self-motions of the fingers, and "degrees of internal force" implies the dimension of the null or "squeeze" forces.

Hint: If A is and $m \times n$ matrix, then Rank(A) + Nullity(A)= n, where Nullity(A) is the dimension of the null space of A.

EXTRA CREDIT: (30 Points)

Determine the rolling contact equations for an ellipse (with major dimensions A, B, and C) rolling on a flat surface. (See Problem #15 on page 262 of the MLS book for example).

You first need to define a coordinate system for the ellipse surface. You could either choose a simple parametrization, such as:

$$g(u, v) = \begin{bmatrix} A\cos(u)\cos(v) \\ B\sin(u)\cos(v) \\ C\sin(v) \end{bmatrix}$$

and convert it into an orthgonal system using the method outlined in the handout on orthogonal coordinate systems, or you could use the following orthgonal parametrization:

$$f(u,v) = \begin{bmatrix} \pm A\sqrt{\frac{(A-u)(A-v)}{(A-B)(A-C)}} \\ \pm B\sqrt{\frac{(B-u)(B-v)}{(B-A)(B-C)}} \\ \pm C\sqrt{\frac{(C-u)(C-v)}{(C-A)(C-B)}} \end{bmatrix}$$

where $u \in [C, B]$, $v \in [B, A]$.