ME 115(b): Final Exam

Instructions

1. Limit your total time to 5 hours. That is, it is okay to take a break in the middle of the exam if you need to ask me a question, or go to dinner, etc.
2. You may use any class notes, books, or other written material.
3. You may use mathematica or any software or computational tools to assist you.
4. Feel free to ask me or the T.A.s questions about the exam.
5. The final is due by 5:00 p.m. on the last day of the final period.
6. The point values are listed for each problem to assist you in allocation of your time.
7. Please put all of your answers in a blue book, or carefully staple your work together in the proper order.

Problem #1 (20 Points)

In class we discussed a variety of "redundancy resolution" techniques. One method which we didn't discuss is the "Augmented Jacobian" technique. In this method, one "augments" the "task vector" by identifying additional "tasks" so that the resulting systems is no longer redundant. For example, let $\vec{x}$ be the location of the end-effector. $\vec{x}$ is related to the manipulator joint angles through the forward kinematics relationship:

$$\vec{x} = f(\vec{\theta})$$

where $\vec{\theta}$ is an n-vector of joint angles and $\vec{x}$ represents the p independent end-effector coordinates. One can define $(n-p)$ additional "tasks":

$$\vec{\phi} = g(\vec{\theta})$$

where $\vec{\phi}$ is the $(n-p) \times 1$ "augmented task vector." For example, one might define elements of ϕ as the orientation or elevation of interior links. Using the augmented task vector, one can then define the "Augmented Jacobian" through the relationship:

$$\begin{bmatrix} \dot{\vec{x}} \\ \dot{\vec{\phi}} \end{bmatrix} = \begin{bmatrix} \mathbf{J} \\ \frac{\partial g(\vec{\theta})}{\partial \vec{\theta}} \end{bmatrix}$$

(a) Describe the conditions under which the Augmented Jacobian loses rank.

(b) Physically interpret each of the conditions in Part (a).

Problem #2: (20 points)

Consider the planar mechanism shown in Figure 1.

1. What is the mobility of this mechanism?
2. Derive the structure equations.
3. Derive the velocity equations.
4. Physically interpret (using your answer to the last question as a guide if necessary) the conditions under which this mechanism becomes singular.

Problem #3: (15 Points)

Let the kinematics of a multi-fingered grasp be modeled by the standard grasp constraint model:

$$J_h(\vec{\theta})\dot{\vec{\theta}} = G^T(\vec{\theta})\vec{V}_{po}^b$$

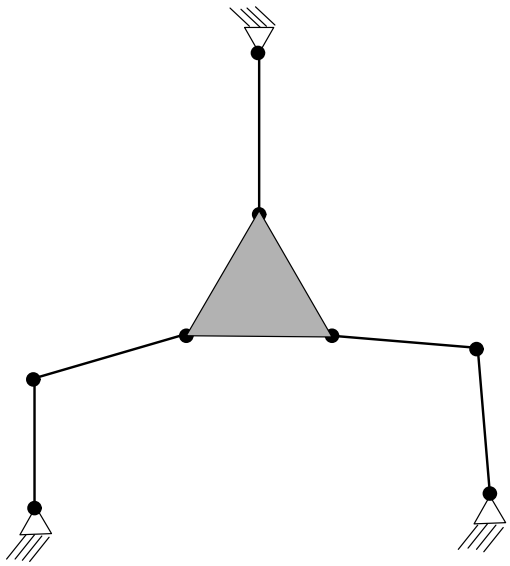

Figure 1: Planar Mechanism

where J_h is the $m \times n$ hand Jacobian matrix, G is the $p \times m$ grasp map (p typically is 3 or 6), and $\vec{\theta}$ is the vector of n finger joint angles. m is the sum of of all the dimensions of the finger wrench bases. $\vec{V}_{po}^{b}$ is the body velocity of the object with respect to a stationary palm frame.

Proposition 5.8 (page 238 of MLS) states that a grasp is *manipulable* if $Range(G^T) \subset Range(J_h)$. However, in class I stated that a grasp is manipulable if, in addition to satisfying Proposition 5.8, the grasp map, G, was full rank.

Assume that a grasp satisfies Prop. 5.8. Discuss the difference in capability of such a grasp if G is full rank, and if G is not full rank. While it is not necessary, you may wish to provide grasp examples which illustrate your discussion.

Problem #4: (25 Points)

Consider the object shown in Figure 2.

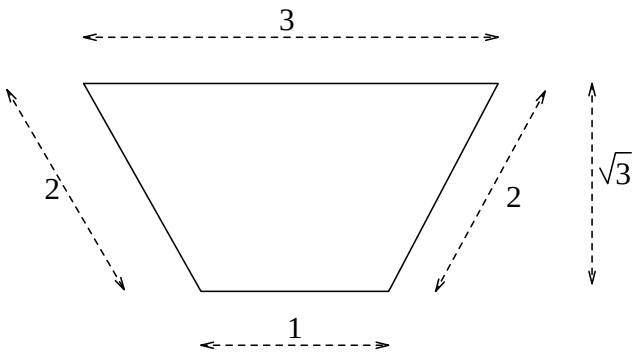

Figure 2: Planar Object

Assume that the object will be grasped by two point contact fingers with friction. Assume that the friction cone half angle is 30^o. Also assume that grasping at the corners is not allowed.

1. Determine the feasible (i.e., force closure) contact regions in the contact configuration space.

Problem #5: (20 Points)

Consider the 3-jointed "cylindrical" robot manipulator shown in Figure 3.

1. Determine the spatial Jacobian Matrix for this mechanism.
2. Determine and physically interpret the singularities of the mechanism.

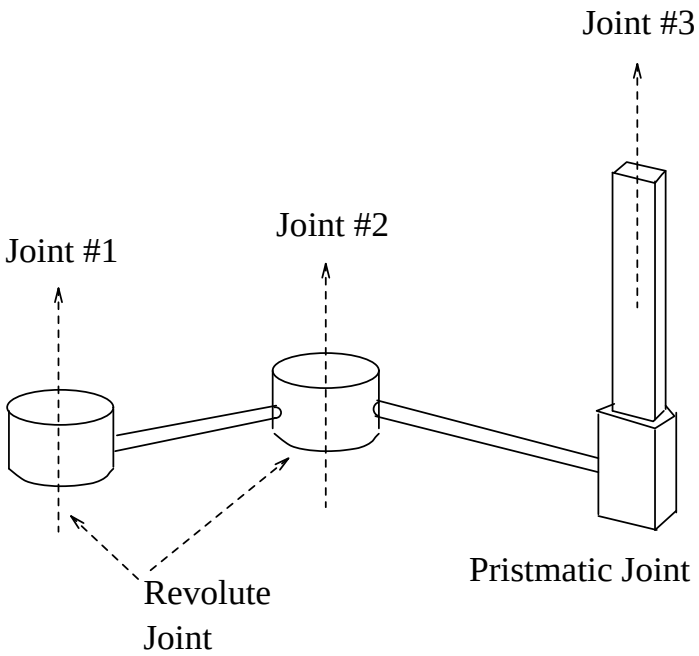

Figure 3: Schematic of a "Cylindrical Manipulator"