ME 115(b): Homework Solution #1

Problem #1: Recall that for this manipulator, the forward kinematics are:

$$g_{ST} = g_{S,1}g_{1,2}g_{2,3}g_{3,T}$$

and that the hybrid Jacobian is:

$$J^h_{ST} = \begin{bmatrix} \frac{\partial p_{ST}}{\partial \theta_1} & \frac{\partial p_{ST}}{\partial \theta_2} & \frac{\partial p_{ST}}{\partial \theta_3} \\ \omega^s_1 & \omega^s_2 & \omega^s_3 \end{bmatrix}$$

where θ_1, θ_2, and θ_3 are the joint variables.

Using the above equation for the forward kinematics, we find that:

$$p_{ST} = \begin{bmatrix} \frac{1}{2}c_1c_2(1+2l+c_3) + \frac{1}{2}l_2c_1s_2(1-c_3) - \frac{l_2}{\sqrt{2}}s_1s_3 \\ \frac{1}{2}s_1c_2(1+2l+c_3) + \frac{1}{2}l_2s_1s_2(1-c_3) - \frac{l_2}{\sqrt{2}}c_1s_3 \\ l_0 - s_2(1+2l+c_3) + \frac{1}{2}l_2c_2(1-c_3) \end{bmatrix}$$

and

$$\frac{\partial p_{ST}}{\partial \theta_1} = \begin{bmatrix} \frac{1}{2}s_1c_2(1+2l+c_3) - \frac{1}{2}l_2s_1s_2(1-c_2) - \frac{l_2}{\sqrt{2}}c_1s_3 \\ \frac{1}{2}c_1c_2(1+2l+c_3) + \frac{1}{2}l_2c_1s_2(1-c_2) - \frac{l_2}{\sqrt{2}}s_1s_3 \\ 0 \end{bmatrix}$$

$$\frac{\partial p_{ST}}{\partial \theta_2} = \begin{bmatrix} -\frac{1}{2}c_1s_2(1+2l+c_3) + \frac{1}{2}l_2c_1c_2(1-c_3) \\ -\frac{1}{2}s_1s_2(1+2l+c_3) + \frac{1}{2}l_2s_1c_2(1-c_3) \\ -\frac{1}{4}(1+2l_1+c_3)l_2c_2s_2(1-c_3) \end{bmatrix}$$

$$\frac{\partial p_{ST}}{\partial \theta_3} = \begin{bmatrix} \frac{1}{2}(-\sqrt{2}l_2c_3s_1 + c_1s_3(l_2s_2 - c_2) \\ \frac{1}{2}(-\sqrt{2}l_2c_3c_1 + s_1s_3(l_2s_2 - c_2) \\ \frac{1}{2}s_3(l_2c_2 + s_2) \end{bmatrix}$$

The twists can be determined by inspection:

$$\omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \omega_2 = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix} \quad \omega_3 = \begin{bmatrix} \frac{1}{\sqrt{2}}c_1 \\ \frac{1}{\sqrt{2}}s_1 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

The next problem (iv) is easier and basically exactly the same. Note, however that the ω_3 corresponding to the prismatic joint is:

$$\omega_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Problem #2: Let g_{ST} denote a rigid body transformation. Recall that the "hybrid" velocity of Frame T is computed from the T's body velocity as:

$$V_{ST}^h = \begin{bmatrix} R_{ST} & 0 \\ 0 & R_{ST} \end{bmatrix} V_{ST}^b$$

where V_{ST}^b is the body velocity of Frame T and R_{ST} is the orientation of Frame T relative to frame S. Recall that the body velocity takes the form:

$$V_{ST}^b = \begin{bmatrix} R_{ST}^T \dot{\vec{p}}_{ST} \\ R_{ST}^T \vec{\omega}_{ST}^s \end{bmatrix}$$

where $\vec{p}_{ST}$ describes the location of Frame T's origin relative to frame S's origin. Hence, the hybrid velocity is:

$$V_{ST}^h = \begin{bmatrix} \dot{\vec{p}}_{ST} \\ \vec{\omega}_{ST}^s \end{bmatrix}$$

and thus the positional part of the hybrid Jacobian is simply derived as

$$\dot{\vec{p}}_{ST} = \frac{\partial \vec{p}_{ST}}{\partial \vec{\theta}} \dot{\vec{\theta}}$$

For manipulator (iii),

$$\vec{p}_{ST} = \begin{bmatrix} (a_2 \cos\theta_2 + a_3 \cos(\theta_2 + \theta_3)) \cos\theta_1 \\ (a_2 \cos\theta_2 + a_3 \cos(\theta_2 + \theta_3)) \sin\theta_1 \\ -(a_2 \sin\theta_2 + a_3 \sin(\theta_2 + \theta_3)) \end{bmatrix}$$

Taking the appropriate derivatives yields:

$$J = \begin{bmatrix} -(a_2 c_2 + a_3 c_{23}) s_1 & -(a_2 s_2 + a_3 s_{23}) c_1 & -a_3 s_{23} c_1 \\ (a_2 c_2 + a_3 c_{23}) c_1 & -(a_2 s_2 + a_3 s_{23}) s_1 & -a_3 s_{23} s_1 \\ 0 & -(a_2 c_2 + a_3 c_{23}) & -a_3 c_{23} \end{bmatrix}$$

where $c_i = \cos\theta_i$, etc.