ME 115(b): Solution to Problem Set #5

Problem #1: Problem 8(a,b) in MLS text.

Part (a):

Attach the following labels to the sides of the object:

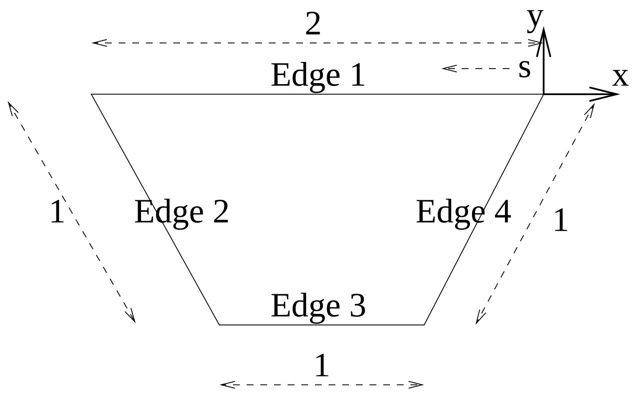

Figure 1: Planar Object, problem 8(a)

Then, there are three unique grasp configurations. For all three cases the constraints will be given by the four inequalities:

$$(n_1 - \mu n_1^\perp) * (p_2 - p_1) > 0$$
$$(n_1 + \mu n_1^\perp) * (p_2 - p_1) < 0$$
$$(n_2 - \mu n_2^\perp) * (p_1 - p_2) > 0$$
$$(n_2 + \mu n_2^\perp) * (p_1 - p_2) < 0$$

where $*$ represents the 2-D cross product, $a * b = det[ab]$. For each case there will be unique expressions for p, n, and $n^\perp$.

Case 1: Grasp edges 1 and 3. Let's place the origin at the center of the object. The parameters are then given by:

$$p_1 = \begin{bmatrix} -1 + s_1 \\ \frac{\sqrt{3}}{4} \end{bmatrix} \quad p_2 = \begin{bmatrix} \frac{-1}{2} + s_2 \\ \frac{-\sqrt{3}}{4} \end{bmatrix}$$
$$n_1 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \quad n_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$n_1^\perp = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad n_2^\perp = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$
$$s_1 \in [0, 2] \quad s_2 \in [0, 1]$$

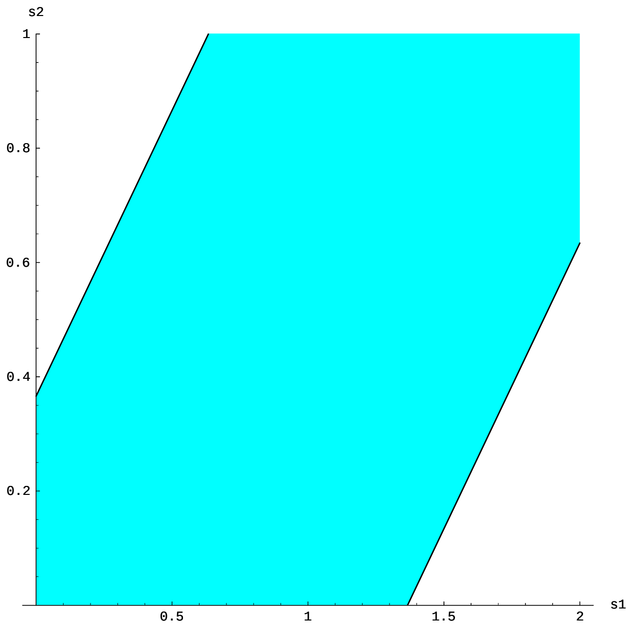

Figure 2: Feasible Grasps, problem 8(a) case 1

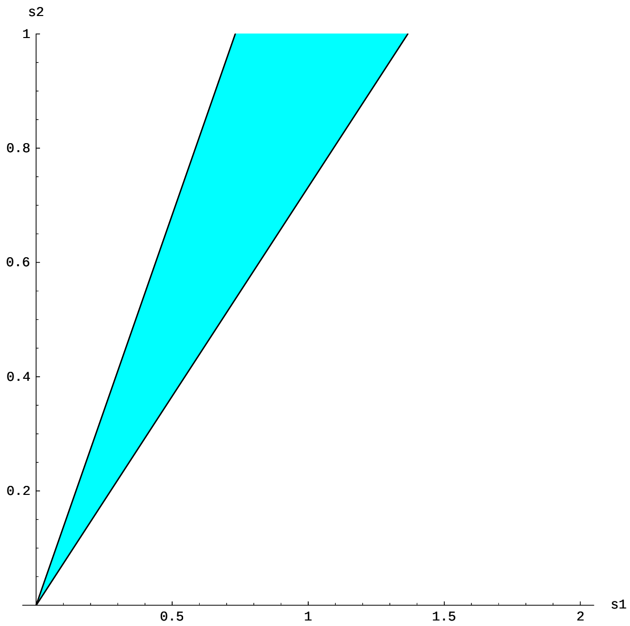

Figure 3: Feasible Grasps, problem 8(a) case 2

Case 2: Grasp edges 1 and 4. Note that this is the same as grasping edges 1 and 2. Let's place the origin where it is shown in the figure above. The parameters are then given by:

$$
\begin{array}{ll}
p_1 = \begin{bmatrix} -s_1 \\ 0 \end{bmatrix} & p_2 = \begin{bmatrix} \frac{-s_2}{2} \\ \frac{-s_2\sqrt{3}}{2} \end{bmatrix} \\
n_1 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} & n_2 = \begin{bmatrix} \frac{-\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} \\
n_1^{\perp} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} & n_2^{\perp} = \begin{bmatrix} \frac{-1}{2} \\ \frac{-\sqrt{3}}{2} \end{bmatrix} \\
s_1 \in [0, 2] & s_2 \in [0, 1]
\end{array}
$$

Case 3: Grasp edges 2 and 4. Let's place the origin at the point below the object where the edges would intersect if they were extended. The parameters are then given by:

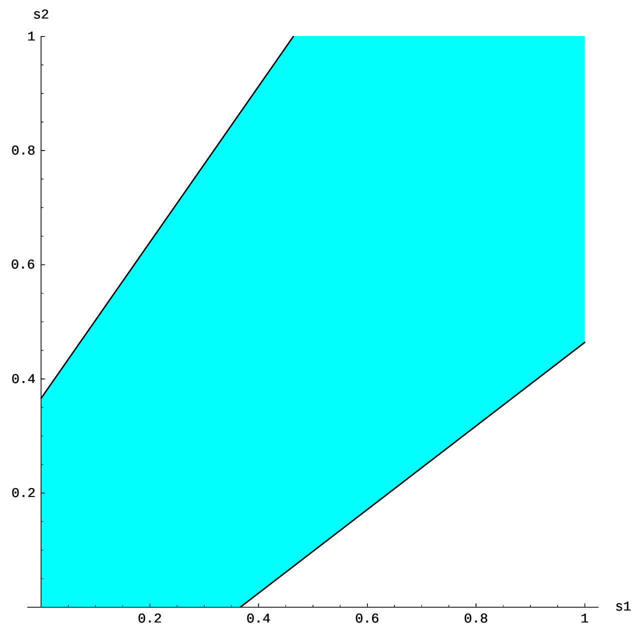

Figure 4: Feasible Grasps, problem 8(a) case 3

$$p_1 = \begin{bmatrix} -\frac{1}{2}(s_1+1) \\ \frac{\sqrt{3}}{2}(s_1+1) \end{bmatrix} \quad p_2 = \begin{bmatrix} \frac{1}{2}(s_2+1) \\ \frac{\sqrt{3}}{2}(s_2+1) \end{bmatrix}$$
$$n_1 = \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} \quad n_2 = \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix}$$
$$n_1^\perp = \begin{bmatrix} \frac{-1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix} \quad n_2^\perp = \begin{bmatrix} \frac{-1}{2} \\ \frac{-\sqrt{3}}{2} \end{bmatrix}$$
$$s_1 \in [0,1] \quad s_2 \in [0,1]$$

Part (b):

The constraint equations will be the same as part (a), using

$$p_1 = \begin{bmatrix} s_1 \\ 0 \end{bmatrix} \quad p_2 = \begin{bmatrix} \frac{s_2}{2} \\ \frac{s_2\sqrt{3}}{2} \end{bmatrix}$$
$$n_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad n_2 = \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{-1}{2} \end{bmatrix}$$
$$n_1^\perp = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad n_2^\perp = \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix}$$
$$s_1 \in [0,1] \quad s_2 \in [0,1]$$

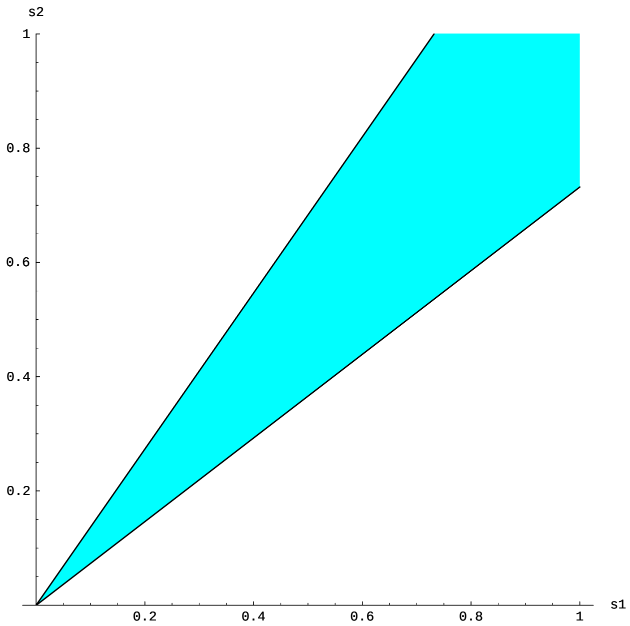

Figure 5: Feasible Grasps, problem 8(b)

The maximal square for each of the above cases can be easily obtained from the plot of the feasible grasps.

Problem #2:

Part 1: Feasible contact regions. These are found the same way as in problem 1 above, using the following parameters.

Case 1, grasping the longer leg and the hypotenuse, with the origin at the intersection of those two sides:

$$
\begin{array}{ll}
p_1 = \begin{bmatrix} -s_1 \\ 0 \end{bmatrix} & p_2 = \begin{bmatrix} \frac{-s_2\sqrt{3}}{2} \\ \frac{-s_2}{2} \end{bmatrix} \\
n_1 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} & n_2 = \begin{bmatrix} \frac{-1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix} \\
n_1^{\perp} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} & n_2^{\perp} = \begin{bmatrix} \frac{-\sqrt{3}}{2} \\ \frac{-1}{2} \end{bmatrix} \\
s_1 \in [0, 1] & s_2 \in [0, \frac{2}{\sqrt{3}}]
\end{array}
$$

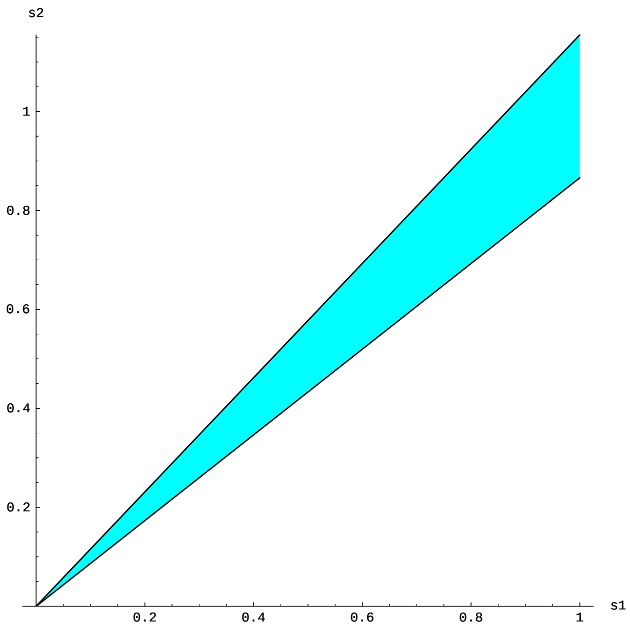

Figure 6: Feasible Grasps, problem 2

Note that for cases 2 and 3, grasping by the short leg and hypotenuse or by the two legs, there are no force-closure grasps.

Part 2: Largest square.

Again, the largest square can be found easily by examining the plot above.

Problem #3:

Part 1: Force closure. The grasp is force closure by inspection, as each contact point lies within the friction cone of the other contact.

Part 2: Manipulability. The grasp is obviously not manipulable. Consider the point where finger #2 makes contact with the object. That point can only translate in a direction perpendicular to finger #2. Therefore, there exist motions of the object which cannot be produced by the grasp.

The grasp constraint equations are of the form:

$$J_h(\theta, x_0)\dot{\theta} = G^T V_{po}^b$$

so, we need to find J_h and G. We'll start with G.

$$G = \begin{bmatrix} Ad^T_{g_{oc_1}^{-1}} B_{c_1} & Ad^T_{g_{oc_2}^{-1}} B_{c_2} \end{bmatrix}$$

Where

$$B_{c_1} = B_{c_2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$g_{oc_1} = \begin{bmatrix} 0 & 1 & -d \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and

$$g_{oc_2} = \begin{bmatrix} 0 & -1 & d \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore,

$$G = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ d & 0 & d & 0 \end{bmatrix}$$

Now, to find J_h, we need B_{c_i} (which we already have), $g_{s_ic_i}$, and $J^s_{s_if_i}$. Calling the angle that the prismatic joint makes with the first link θ_0, we have

$$J^s_{s_1 f_1} = \begin{bmatrix} 0 & \cos(\theta_1+\theta_0) \\ 0 & \sin(\theta_1+\theta_0) \\ 1 & 0 \end{bmatrix}$$

and,

$$J^s_{s_2 f_2} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Also, calling the position and orientation of the object frame with respect to the base stationary frame (x_0, y_0, ϕ), we have that

$$g_{s_1 c_1} = \begin{bmatrix} \sin\phi & \cos\phi & B + x_0 \\ -\cos\phi & \sin\phi & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

and,

$$g_{s_2 c_2} = \begin{bmatrix} -\sin\phi & -\cos\phi & -B + x_0 \\ \cos\phi & -\sin\phi & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

Using these, and the equation for J_h:

$$J_h = \begin{bmatrix} B^T_{c_1} Ad^{-1}_{g_{s_1 c_1}} J^s_{s_1 f_1} & 0 \\ 0 & B^T_{c_2} Ad^{-1}_{g_{s_2 c_2}} J^s_{s_2 f_2} \end{bmatrix}$$

gives the expression:

$$J_h = \begin{bmatrix} -(B+x_0)\cos\phi - y_0\sin\phi & \sin\phi\cos(\theta_1+\theta_0) - \cos\phi\sin(\theta_1+\theta_0) & 0 \\ (B+x_0)\sin\phi - y_0\cos\phi & \cos\phi\cos(\theta_1+\theta_0) + \sin\phi\sin(\theta_1+\theta_0) & 0 \\ 0 & 0 & -(B+x_0)\cos\phi + y_0\sin\phi \\ 0 & 0 & (B+x_0)\sin\phi + y_0\cos\phi \end{bmatrix}$$

Thus, we have all the terms for the grasp constraint equation,

$$J_h(\theta, x_0)\dot{\theta} = G^T V^b_{po}$$