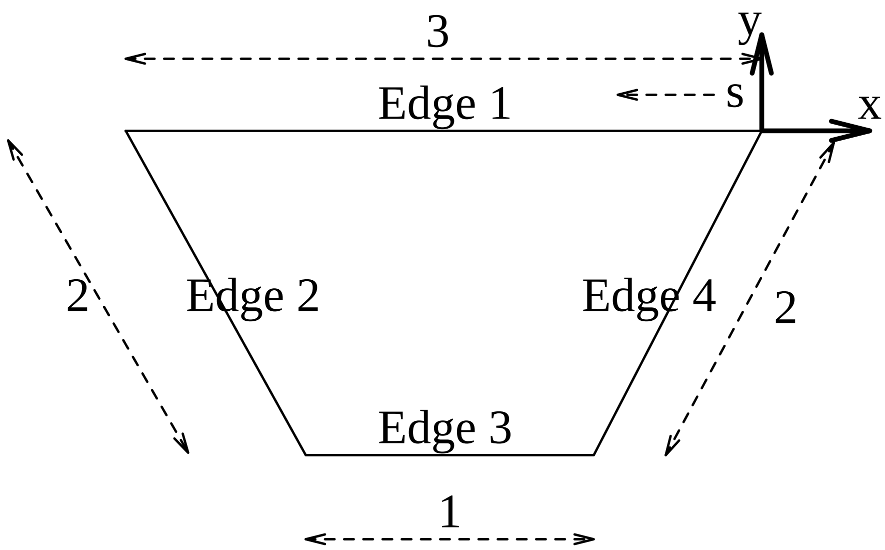

Problem #2

Attach the following labels to the sides of the object:

Let $s = 0$ denote the contact location on the upper right vertex. Further, let us measure increasing s in a clockwise direction. Thus, $s \in [0, 8]$. Similarly, attach a reference coordinate frame to the upper right vertex.

There are 6 possible pairs of edges to check to determine the feasible force closure regions: pairs 1&2, 1&3, 1&4, 2&3, 2&4, and 3&4. Let $\vec{n}_i$ and $\vec{n}_j$ be the normals to edges i and j. Likewise, let t_i and t_j be the tangents to edges i and j. Let $\vec{p}_i = \vec{p}(s_i)$ be the location of the contact point on the i^{th} edge, as measured in the body fixed reference frame. Likewise, let $\vec{p}_j = \vec{p}(s_j)$ denoted the location of the second contact on the j^{th} edge.

The force closure conditions for a pair of contacts on edges i and j are:

$$\begin{aligned}
(\vec{n}_i + \mu\vec{t_i}) \times [\vec{p}(s_i) - \vec{p}(s_j)] &< 0 \\
(\vec{n}_i - \mu\vec{t_i}) \times [\vec{p}(s_i) - \vec{p}(s_j)] &> 0 \\
(\vec{n}_j + \mu\vec{t_j}) \times [\vec{p}(s_j) - \vec{p}(s_i)] &< 0 \\
(\vec{n}_j - \mu\vec{t_j}) \times [\vec{p}(s_j) - \vec{p}(s_i)] &> 0
\end{aligned} \tag{1}$$

where μ is the coefficient of friction, which is assumed to be $\mu = \tan(30^o) = 1/\sqrt{3}$ in this problem.

We can now evaluate the conditions on the 6 pairs of edges:

Edges 1&2: There are no feasible regions.

Edges 1&3:

Edges 1&4: No feasible regions.

Edges 2&3: No feasible regions.

Edges 2&4: No feasible regions.

Edges 3&4: No feasible regions.

Problem #3

- (a) False. A counter-example is given below. In this counter-example, if the contacts are PCNF, then the grasp is manipulable, but not force closure.
- (b) True. Let $N(A)$ and $R(A)$ respectively denote the null and range spaces of a matrix A. A sketch of this proof goes as follows
  - Recall that manipulability implies that $R(G^T) \subset R(J_h)$, where G is the grasp map, and J_h is the hand Jacobian of the multi-fingered grasp.
  - Also recall that $R(J_h) + N(J_h^T) = \mathbb{R}^m$, where m is the total number of finger joints. In other words, $R(J_h)$ and $N(J_h^T)$ are orthogonal spaces, and their direct sum spans an m-dimensional vector space.
  - The existence of statically indeterminate forces implies that: $N(J_h^T) \cap R(G^T) \neq \emptyset$. That is, statically indeterminate forces are those finger contact forces which are in the null space of J_h^T and which do not lead to "squeeze" forces.
  - However, if $R(G^T) \subset R(J_h)$, then elements in $R(G^T)$ can not be in $N(J_h^T)$, and thus there can be no statically indeterminate forces.

Problem #4:

Part (a): Yes, the grasp is force closure, since planar two-fingered antipodal point PCWF grasps are force closure.

Part (b): The relevant quantities required to compute the grasp map are:

$$\textbf{wrench basis}: \; B_1 = B_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$g_{oc_1} = \begin{bmatrix} 0 & 1 & -d \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad g_{oc_2} = \begin{bmatrix} 0 & -1 & d \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$G = [G_1 \mid G_2] = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ d & 0 & d & 0 \end{bmatrix}$$

The spatial Jacobians of the two fingers are:

$$J^s_{s_1 f_1} = \begin{bmatrix} 0 & (l_1 s_1) & (l_1 s_1 + l_2 s_{12}) \\ 0 & -(l_1 c_1) & -(l_2 c_1 + l_2 c_{12}) \\ 1 & 1 & 1 \end{bmatrix} \qquad J^2_{s_2 f_2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

where $s_1 = \sin(\theta_1)$, $s_{12} = \sin(\theta_1 + \theta_2)$, etc.

The grasp constraint equations can be put in the form:

$$J_h \dot{\vec{\theta}} = G^T \xi^b_{po}$$

where ξ^b_{po} is the (body) velocity of the object, and J_h is the hand Jacobian, where:

$$J_h = \begin{bmatrix} B_1^T Ad^{-1}_{g_{s_1 c_1}} J^s_{s_1 f_1} & 0 \\ 0 & B_2^T Ad^{-1}_{g_{s_2 c_2}} J^s_{s_2 f_2} \end{bmatrix}$$

B_1 and B_2 are as above. The transformation $Ad^{-1}_{g_{s_i c_i}}$ can be expressed as:

$$Ad^{-1}_{g_{s_i c_i}} = (Ad_{g_{s_i f_i}} Ad_{g_{f_i c_i}})^{-1} = Ad^{-1}_{g_{f_i c_i}} Ad^{-1}_{g_{s_i f_i}}$$

In this case,

$$Ad_{g_{s_i f_i}} = \begin{bmatrix} R_{s_i f_i} & S(p_{s_i f_i}) R_{s_i f_i} \\ 0 & 1 \end{bmatrix}$$

where:

$$R_{s_1 f_1} = \begin{bmatrix} c_{123} & -s_{123} \\ s_{123} & c_{123} \end{bmatrix} \qquad R_{s_2 f_2} = \begin{bmatrix} c_4 & -s_4 \\ s_4 & c_4 \end{bmatrix}$$

$$p_{s_1 f_1} = \begin{bmatrix} l_1 c_1 + l_2 c_{12} + l_3 c_{123} \\ l_1 s_1 + l_2 s_{12} + l_3 s_{123} \end{bmatrix} \qquad p_{s_2 f_2} = \begin{bmatrix} l_4 c_4 \\ l_4 s_4 \end{bmatrix}$$

$Ad_{g_{f_i c_i}}$ corresponds to the amount of rotation of the i^{th} finger frame required to bring it in alignment with the i^{th} contact frame. The amount of rotation will depend upon the choice of a reference orientation for the body.

The hand is obviously *not* manipulable, as the right hand finger cannot follow any motion which takes the contact point out of a circular arc.