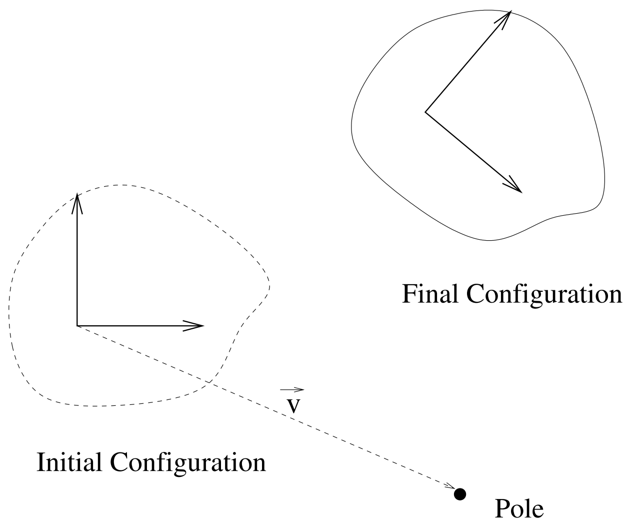
ME 115(a): Homework #1

(Due Friday January 20, 2006)

Problem 1: (10 points) Every planar rigid body displacement is *equivalent* to a rotation about a unique point in the plane, known as the pole (see Figure).



Let A be a fixed reference frame. A rigid body, L, which has local frame B attached to it, is located relative to reference frame A by $D_1 = (\vec{d}_{01}, R_{01},)$. Body L moves to position C, where the displacement to location C, as measured by an observer in frame B, is given by $D_2 = (\vec{d}_{12}, R_{12})$. Where is the *pole* of the body displacement from position B to position C, as a function of R_{01}, R_{12}, $\vec{d}_{01}$, and $\vec{d}_{12}$?

a. As measured in Frame A

b. As measured in Frame B

c. As measured in Frame C

Problem 2: (5 points) In the above problem, suppose $D_1 = (x, y, \theta) = (1.0, 2.0, 30.0^o)$ and $D_2 = (x, y, \theta) = (2.0, 2.0, 45^o)$. Where is the pole of the displacement from B to C in this case?

Problem 3: (15 points) Using the set up of Problem 1, pick a coordinate system whose origin is located at the pole of the displacement, and show that in this coordinate system, the displacement of the body from B to C is a pure rotation.

Problem 4: (10 points) (This is Problem 3(c) in Chapt. 2 of the MLS text). Let $R = [\vec{r}_1 \ \vec{r}_2 \ \vec{r}_3]$ be a rotation matrix (i.e., the $\vec{r}_i$ are the columns of the matrix). Show that $det(R) = \vec{r}_1^T(\vec{r}_2 \times \vec{r}_3)$

Problem 5: (15 points) A *planar reflection* is an operation wherein one "reflects" all of the particles in a body across a line. Show (intuitively) that reflections "preserve length." That is, reflections do not alter the distance relationship between particles in a rigid body. Can any planar displacement be equivalently performed by a reflection?

Problem 6: (10 points) In class we used the particle nature of rigid bodies to "prove" that a planar rigid body has three degrees of freedom. Use the same idea to "prove" that a body undergoing spherical motion has three degrees of freedom.