

## ME 115(a): Homework #1

(Due Friday January 15, 2010)

**Problem 1:** (10 points) Let  $\mathcal{F}_1$  denote a fixed reference frame in the plane, with orthonormal basis vectors  $\vec{x}_1$  and  $\vec{y}_1$ . Similarly, consider a second reference frame  $\mathcal{F}_2$  with orthonormal basis vectors  $\vec{x}_2$  and  $\vec{y}_2$ . Let  $d_{12} = [x \ y]^T$  be the vector pointing from the origin of  $\mathcal{F}_1$  to the origin of  $\mathcal{F}_2$ . Let  $\theta_{12}$  denote the relative orientation of the two reference frames:  $\theta_{12}$  is the angle between  $\vec{x}_1$  and  $\vec{x}_2$ . Let  ${}^2\vec{v} = [{}^2v_x \ {}^2v_y]^T$  denote the coordinates of a point,  $P$ , as seen by an observer in  $\mathcal{F}_2$ . In class we developed a formula for the coordinate transformation of  $P$  to its representation in  $\mathcal{F}_1$ :

$${}^1\vec{v} = \vec{d}_{12} + R(\theta_{12}) {}^2\vec{v} \quad (1)$$

where  $R(\theta_{12})$  is the  $2 \times 2$  rotation matrix:

$$R(\theta_{12}) = \begin{bmatrix} \cos \theta_{12} & -\sin \theta_{12} \\ \sin \theta_{12} & \cos \theta_{12} \end{bmatrix}$$

For computational purposes, it is sometimes convenient to use different representations of coordinates, vectors, and rotations. For example, consider complex numbers such that if  $\vec{w}$  is a  $2 \times 1$  vector  $\vec{w} = [w_1 \ w_2]^T$  in the plane, then  $\tilde{w} = w_1 + iw_2$  where  $i$  is the complex number such that  $i \cdot i = -1$ . Show that if  ${}^2\tilde{v}$  is the complex representation of  ${}^2\vec{v}$ , then the complex representation of the coordinate transform in Equation (1) is:

$${}^1\tilde{v} = \tilde{d}_{12} + e^{i\theta_{12}} {}^2\tilde{v}$$

**Problem 2:** (10 points) Every planar rigid body displacement is *equivalent* to a rotation about a unique point in the plane, known as the pole (see Figure 1).

Let A be a fixed reference frame. A rigid body, L, which has local frame B attached to it, is located relative to reference frame A by  $D_1 = (\vec{d}_{01}, R_{01})$ . Body L moves to position C, where the displacement to location C, as measured by an observer in frame B, is given by  $D_2 = (\vec{d}_{12}, R_{12})$ . Where is the *pole* of the body displacement from position B to position C, as a function of  $R_{01}$ ,  $R_{12}$ ,  $\vec{d}_{01}$ , and  $\vec{d}_{12}$ ?

- As measured in Frame A
- As measured in Frame B
- As measured in Frame C

**Problem 3:** (5 points) In the above problem, suppose  $D_1 = (x, y, \theta) = (1.0, 2.0, 30.0^\circ)$  and  $D_2 = (x, y, \theta) = (2.0, 2.0, 45^\circ)$ . Where is the pole of the displacement from B to C in this case?

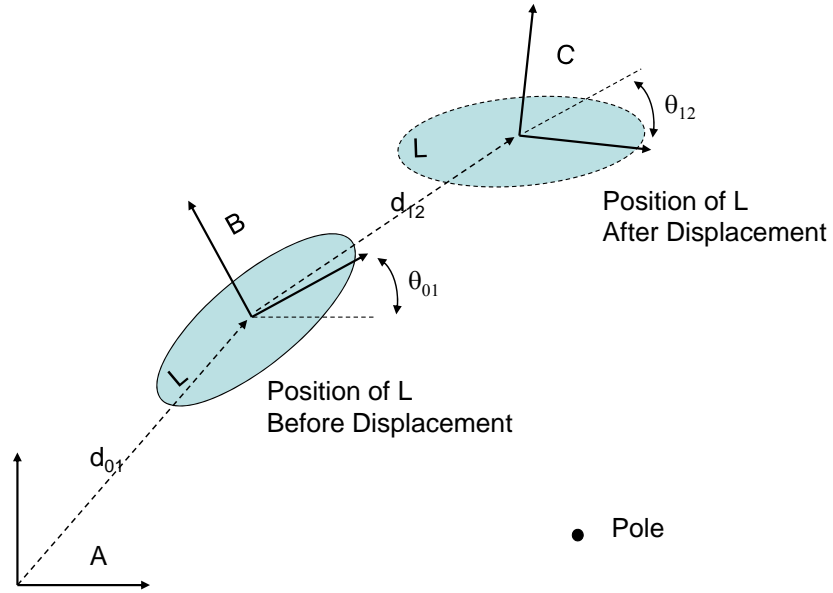


Figure 1: Geometry of planar displacement

**Problem 4:** (15 points) Using the set up of Problem 2, pick a coordinate system whose origin is located at the pole of the displacement, and show that in this coordinate system, the displacement of the body from B to C is a pure rotation.

**Problem 5:** (15 points) A *planar reflection* is an operation wherein one “reflects” all of the particles in a body across a line. Show (intuitively) that reflections “preserve length.” That is, reflections do not alter the distance relationship between particles in a rigid body. Can any planar displacement be equivalently performed by a reflection?