Problem 1: (Special Configurations of “Slider-Crank” linkages). Consider the “slider-crank” four bar linkage shown Figure 1. This is the piston mechanism in an internal combustion engine. Unlike the four bar linkages we have studied so far, it consists of three revolute joints and one prismatic joint.

This problem considers the stationary configurations of this mechanism. One of the special configurations is obvious: the piston (or prismatic joint) is at “top dead center,” and the cylinder comes momentarily to rest in this position. To show that this is indeed a special configuration, perform the following calculations/analysis:

- derive the twist coordinates for the joint axis screws.
- It was shown in class that the special (or stationary) configurations for the $j^{th}$ joint occur when the determinant of the cofactor of the $jj$ element of the grammian matrix:

\[
\begin{vmatrix}
\xi_1 \cdot \xi_1 & \xi_1 \cdot \xi_2 & \xi_1 \cdot \xi_3 & \xi_1 \cdot \xi_4 \\
\xi_2 \cdot \xi_1 & \xi_2 \cdot \xi_2 & \xi_2 \cdot \xi_3 & \xi_2 \cdot \xi_4 \\
\xi_3 \cdot \xi_1 & \xi_3 \cdot \xi_2 & \xi_3 \cdot \xi_3 & \xi_3 \cdot \xi_4 \\
\xi_4 \cdot \xi_1 & \xi_4 \cdot \xi_2 & \xi_4 \cdot \xi_3 & \xi_4 \cdot \xi_4
\end{vmatrix}
\]

becomes zero (where $\xi_j$ denotes the twist for joint axis $j$).

- (a) Show that joint 4 has a stationary configuration when $\theta_1 = 0$ and $\theta_2 = 0$.
- (b) Develop an expression for the stationary configurations of joint 1. What are the necessary conditions for joint 1 to have a stationary configuration?

Problem 2: Problem 21, Chapter 3 of MLS.