

ME 115(b): Homework #2
(Due Wednesday, April 23, 2014)

Problem 1: This problem is almost entirely computational. You can program in your favorite computer language.

Consider the three link planar robot shown in Figure 1

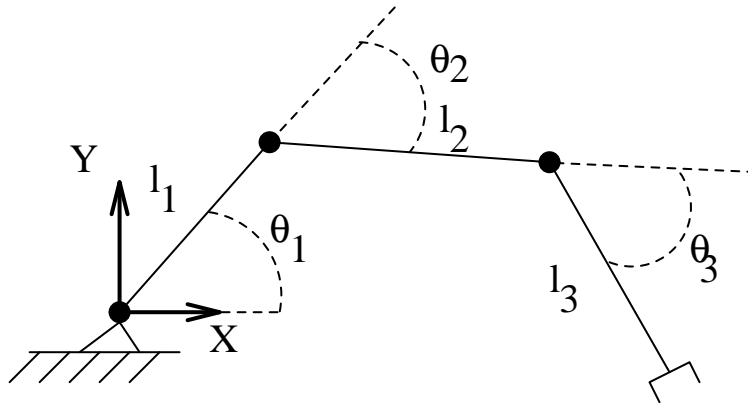


Figure 1: 3R planar “redundant” robot

This robot has three revolute joints. If we are only concerned with positioning the end-effector in the plane, and not concerned about the orientation of the last link, then this robot is redundant. It is the simplest redundant robot known, and often used as a toy example. Assume that $l_1 = l_2 = l_3 = 1$.

- (a) Compute the forward kinematics, $f(\vec{\theta})$, of this manipulator (in symbolic form). I.e., $f(\vec{\theta})$ describes the location of the tool frame origin with respect to the origin of the stationary frame.
- (b) Compute the “hybrid” Jacobian matrix of this manipulator (in symbolic form). That is, compute $\partial f(\vec{\theta}) / \partial \vec{\theta}$.
- (c) Let the desired end-effector trajectory be a circle centered at $(x, y) = (1.5, 0.0)$, with a radius of 0.5. One such way to parametrize this circle is:

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 1.5 - 0.5 \cos(\omega t) \\ 0.0 - 0.5 \sin(\omega t) \end{bmatrix}$$

where ω controls the speed of the trajectory. Pick ω to be a reasonable number, such as π , or $\pi/2$.

Simulate this robot following the circular trajectory using the resolved rate trajectory planning scheme with a simple pseudo-inverse solution:

$$\dot{\theta} = J^\dagger(\vec{\theta}) \dot{\vec{x}}(t)$$

where J^\dagger is the Moore-Penrose pseudo-inverse of the hybrid Jacobian J , and where $\dot{\vec{x}}(t)$ is derived from the circular trajectory. You can use simple Euler integration to integrate the pseudo-inverse, or savvy mathematica hackers can use Mathematica's *NDSolve* function to integrate the equations.

The output of your simulation should be:

- (1) A plot of the joint angles versus time.
 - (2) A plot of the actual end-effector position versus time.
- (d) (extra credit) Increase the radius of the above trajectory circle to 2.99 (so that it nearly touches the workspace boundary). Repeat the above simulation. Now repeat the above simulation using a damped pseudo-inverse solution.