ME 115(a): Homework #3

(Due Thursday Feb. 7, 2002)

(Note: this is a corrected version of the homework handed out in class, with the unnecessary problems removed)

Problem 1: Do Problem 6(a,b,d) in Chapter 2 of MLS.

Problem 3: Do Problem 10(b,c) in Chapter 2 of the MLS text. For part (b), you do not need to answer the part about surjectivity.

Problem 5: Consider 2×2 complex matrices of the form:

$$M = \begin{bmatrix} z & w \\ -w^* & z^* \end{bmatrix} = \begin{bmatrix} (a+ib) & (c+id) \\ -(c-id) & (a-ib) \end{bmatrix}$$

where:

$$det(M) = zz^* + ww^* = 1$$

and $z, w \in \mathbb{C}$, and $*$ denotes complex conjugation. Such matrices form a matrix group termed the "special unitary matrices" of dimension 2, $SU(2)$.

- **Part (a):** Show that matrices:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} \quad \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$$

  form a basis for $SU(2)$. The element i is $\sqrt{-1}$. I.e., all elements of $SU(2)$ can be expressed as some combination of these elements. Next show that elements of $SU(2)$ are isomorphic to the unit quaternions. That is, there is a one-to-one correspondence between each element of $SU(2)$ and a unit quaternion.

- **Part (b):** Show that the special unitary representation of a rotation in terms of z-y-x Euler Angles can be computed as :

$$\begin{bmatrix} \cos\frac{\psi}{2} & i\sin\frac{\psi}{2} \\ i\sin\frac{\psi}{2} & \cos\frac{\psi}{2} \end{bmatrix} \begin{bmatrix} \cos\frac{\phi}{2} & \sin\frac{\phi}{2} \\ -\sin\frac{\phi}{2} & \cos\frac{\phi}{2} \end{bmatrix} \begin{bmatrix} e^{i\frac{\gamma}{2}} & 0 \\ 0 & e^{-i\frac{\gamma}{2}} \end{bmatrix}$$

  where ψ, ϕ, and γ are respectively the rotations about the z, y, and x axes.

- **Part (c):** Suppose a rotation is represented by a special unitary matrix of the form:

$$\begin{bmatrix} z & w \\ -w^* & z^* \end{bmatrix}$$

  determine the angle of rotation, ϕ, and the axis of rotation, $\hat{s}$.