ME 115(a): Homework #3

(Due Thursday October 28, 1999)

Problem 1: Do Problem 6(b,d) in Chapter 2 of the MLS text.

Problem 2 Do Problem 10(b,c) in Chapter 2 of the MLS text.

Problem 3 Do Problem 11(a,b) in Chapter 2 of the MLS text.

Problem 4: (Rigid Body Spatial Displacement) A rigid body undergoes spatial displacement. Before displacement, three points in the body are:

$$P_0 = (1.0, 0.0, 0.0) \qquad Q_0 = (1.0, 1.0, 0.0) \qquad R_0 = (1.0, 0.0, 1.0)$$

After displacement the above points are now located at:

$$\begin{aligned} P_1 &= (2.66667, 2.66667, 1.66667) \\ Q_1 &= (2.33333, 3.33333, 2.33333) \\ R_1 &= (3.33333, 2.33333, 2.33333) \end{aligned}$$

1. Compute the screw displacement parameters: $\vec{\omega}$ (a unit vector collinear with the screw axis), ϕ (rotation about the screw axis), $\vec{\rho}$ (a line from the origin to a point on the screw axis), and $d^{||}$ (the amount of translation along the screw axis).
2. Compute the rotation matrix and displacement vector of this spatial displacement.

Problem 5: Consider 2×2 complex matrices of the form:

$$M = \begin{bmatrix} z & w \\ -w^* & z^* \end{bmatrix} = \begin{bmatrix} (a+ib) & (c+id) \\ -(c-id) & (a-ib) \end{bmatrix}$$

where:

$$det(M) = zz^* + ww^* = 1$$

and $z, w \in \mathbb{C}$, and $*$ denotes complex conjugation. Such matrices form a matrix group termed the "special unitary matrices" of dimension 2, $SU(2)$. We showed in class that elements of $SU(2)$ are isomorphic to the unit quaternions. That is, there is a one-to-one correspondence between each element of $SU(2)$ and a unit quaternion.

- **Part(a):** Show that the special unitary representation of a rotation in terms of z-y-x Euler Angles can be computed as :

$$\begin{bmatrix} \cos\frac{\psi}{2} & i\sin\frac{\psi}{2} \\ i\sin\frac{\psi}{2} & \cos\frac{\psi}{2} \end{bmatrix} \begin{bmatrix} \cos\frac{\phi}{2} & \sin\frac{\phi}{2} \\ -\sin\frac{\phi}{2} & \cos\frac{\phi}{2} \end{bmatrix} \begin{bmatrix} e^{i\frac{\gamma}{2}} & 0 \\ 0 & e^{-i\frac{\gamma}{2}} \end{bmatrix}$$

  where ψ, ϕ, and γ are respectively the rotations about the z, y, and x axes.

- **Part (b):** Suppose a rotation is represented by a special unitary matrix of the form:

$$\begin{bmatrix} z & w \\ -w^* & z^* \end{bmatrix}$$

  determine the angle of rotation, ϕ, and the axis of rotation, $\hat{s}$.