ME 115(a): Homework #4

(Due Friday, February 12, 2010)

Problem 1:(10 points). Assume that the orientation of a rigid body is described by z-y-z Euler angles, where the angles of rotation are respectively ψ, ϕ, and γ. Further assume that the body is spinning with rotation rates of $\dot{\psi}$, $\dot{\phi}$, and $\dot{\gamma}$ about the respective z, y, and z axes. Show that the spatial angular velocity of the body is:

$$\vec{\omega}^s = \begin{bmatrix} -\dot{\phi}\sin\psi + \dot{\gamma}\cos\psi\sin\phi \\ \dot{\phi}\cos\psi + \dot{\gamma}\sin\psi\sin\phi \\ \dot{\psi} + \dot{\gamma}\cos\phi \end{bmatrix}$$

Note that the solution to this problem is useful for the study of gyroscopes.

Problem 2: (15 points) Problem 11(a,b,e) in Chapter 2 of MLS.

Problem 3: (10 points) Problem 13 in Chapter 2 of MLS.

Problem 4: (10 points) Problem 14 in Chapter 2 of MLS.

Problem 5: (15 points) Do Problem 18(a,b,c,d) in Chapter 2 of MLS.

Problem 6: (20 points). Let B be a rigid body whose motions can be described by $g(t) \in SE(3)$, which measures the location of a frame fixed to the moving body with respect to a stationary frame. Assume that points P, Q, and R are fixed in this moving body. Further assume that the coordinates of these points, as seen by an observer in a frame affixed to the body, are $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$. Assume that at a given instant of time, t_0, we can measure both the location of the points P, Q, and R, as well the velocities of these points, $\dot{P}$, $\dot{Q}$, and $\dot{R}$— as seen by an observer in the stationary frame. Show how to compute either the spatial or the body velocity of B from this information.

The solution to this problem is at the foundation of some "motion capture" algorithms.