ME 115(b): Problem Set #5
(Due Wednesday, May 17, 2006)

Problem #1

Let $\alpha\colon I \to \mathbb{R}^2$ be a regular parametrized plane curve. The curve:

$$\beta(t) = \alpha(t) + \frac{1}{\kappa(t)}\vec{n}(t)$$

is known as the *evolute* of $\alpha(t)$. $\kappa(t)$ is the curvature at t, while $\vec{n}$ is the unit normal vector at t. Evolutes are important in gear theory.

(a) Show that the tangent at t of the evolute of $\alpha(t)$ is the normal to α at t.

(b) what is the evolute of a circle?

Problem #2:

Derive the contact equations for a planar ellipse rolling/sliding on a circle of radius R. Recall that the boundary of a planar ellipse can be parametrized as:

$$\begin{bmatrix} x(u) \\ y(u) \end{bmatrix} = \begin{bmatrix} a\cos(u) \\ b\sin(u) \end{bmatrix}$$

Note that when $a = b$, the ellipse is a circle. Under what conditions does the relative curvature become ill defined?