ME 115(a): Solution to Homework #2

Problem #1:

Part (a): You can find the forward kinematics of this manipulator by using Denavit Hartenberg paramaters, or by product of exponentials.

$$g_{01} = \begin{bmatrix} cos\theta_1 & -sin\theta_1 & 0 & 0 \\ sin\theta_1 & cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{12} = \begin{bmatrix} cos\theta_2 & -sin\theta_2 & 0 & l_1 \\ sin\theta_2 & cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{23} = \begin{bmatrix} cos\theta_3 & -sin\theta_3 & 0 & l_2 \\ sin\theta_3 & cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{12} = \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{ST} = g_{01} g_{12} g_{23} g_{34} \tag{1}$$

Plugging in the g values,:

$$g_{ST} = \begin{bmatrix} cos(\theta_1+\theta_2+\theta_3) & -sin(\theta_1+\theta_2+\theta_3) & 0 & l_1 cos(\theta_1) + l_2 cos(\theta_1+\theta_2) + l_3 cos(\theta_1+\theta_2+\theta_3) \\ sin(\theta_1+\theta_2+\theta_3) & cos(\theta_1+\theta_2+\theta_3) & 0 & l_1 sin(\theta_1) + l_2 sin(\theta_1+\theta_2) + l_3 sin(\theta_1+\theta_2+\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus,

$$R_{ST} = \begin{bmatrix} cos(\theta_1+\theta_2+\theta_3) & -sin(\theta_1+\theta_2+\theta_3) & 0) \\ sin(\theta_1+\theta_2+\theta_3) & cos(\theta_1+\theta_2+\theta_3) & 0) \\ 0 & 0 & 1 \end{bmatrix}$$

$$p_{ST} = \begin{bmatrix} l_1cos(\theta_1) + l_2cos(\theta_1+\theta_2) + l_3cos(\theta_1+\theta_2+\theta_3) \\ l_1sin(\theta_1) + l_2sin(\theta_1+\theta_2) + l_3sin(\theta_1+\theta_2+\theta_3) \\ 0 \end{bmatrix}$$

Part (b): The Hybrid Jacobian is defined as

$$J^H = \begin{bmatrix} \frac{dp}{d\theta} \\ (\frac{dR}{d\theta}R^{(}-1))^V \end{bmatrix}$$

Now using the solutions from part (a):

$$J^H = \left[\begin{matrix} -sin\theta_1 l_1 - sin(\theta_1+\theta_2)l_2 - sin(\theta_1+\theta_2+\theta_3)l_3 & -sin(\theta_1+\theta_2)l_2 - sin(\theta_1+\theta_2+\theta_3)l_3 & -sin(\theta_1+\theta_2+\theta_3 \\ -cos\theta_1 l_1 - cos(\theta_1+\theta_2)l_2 - cos(\theta_1+\theta_2+\theta_3)l_3 & cos(\theta_1+\theta_2)l_2 + cos(\theta_1+\theta_2+\theta_3)l_3 & cos(\theta_1+\theta_2+\theta_3)l \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{matrix}\right.$$

Part (c): Now we can use the fact that all of the l values are 1. Thus,

$$J^H = \begin{bmatrix} -sin\theta_1 - sin(\theta_1+\theta_2) - sin(\theta_1+\theta_2+\theta_3) & -sin(\theta_1+\theta_2) - sin(\theta_1+\theta_2+\theta_3) & -sin(\theta_1+\theta_2+\theta_3) \\ -cos\theta_1 - cos(\theta_1+\theta_2) - cos(\theta_1+\theta_2+\theta_3) & cos(\theta_1+\theta_2) + cos(\theta_1+\theta_2+\theta_3) & cos(\theta_1+\theta_2+\theta_3) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

We are given that

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 1.5 - 0.5cos(\omega t) \\ 0.0 - 0.5sin(\omega t) \end{bmatrix}$$

where ω controls the speed of the trajectory. The value of ω is left up to you, with the suggestion of π or $\frac{\pi}{2}$.

You are asked to simulate the robot following a circular trajectory using the resolved rate trajectory planning scheme with a simple pseudo inverse solution:

$$\dot{\Theta} = J^{\dagger}\theta\dot{p} \tag{2}$$

Remember that the pseudo-inverse can be found with the following equation:

$$J^{\dagger} = J^T(J^T)^{-1} \tag{3}$$

The actual coding of this will depend on whether you are using Mathematica, Matlab, or another package.