ME 115(b): Solution to Problem Set #3

Problem #1: Problem 21(a,b,c,e) in MLS text.
Part (a.) The number of degrees of freedom of the mechanism, F, is given by the planar version of Gruebler's formula. Now g= 9 joints, f = 1 DOF/link, and N = 7 links.

Thus,

$$F = 3(7 - 9) + 9 \times 1 = 3 \tag{1}$$

Part (b.) Define the zero configuration of the mechanism. We have chosen it to be the left picture in the problem statement. The structure equation of the mechanism has the form

$$g_{st} = e^{\hat{\xi}_{11}} e^{\hat{\xi}_{12}} e^{\hat{\xi}_{13}} g_{st}(0) = e^{\hat{\xi}_{21}} e^{\hat{\xi}_{22}} e^{\hat{\xi}_{23}} g_{st}(0) = e^{\hat{\xi}_{31}} e^{\hat{\xi}_{32}} e^{\hat{\xi}_{33}} g_{st}(0) \tag{2}$$

where

$$\xi_{11} = \begin{bmatrix} \frac{h}{2} \\ 0 \\ 1 \end{bmatrix}$$

$$\xi_{12} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\xi_{13} = \begin{bmatrix} \frac{h}{2} \\ -\omega \\ 1 \end{bmatrix}$$

$$\xi_{21} = \begin{bmatrix} frach2 \\ 0 \\ 1 \end{bmatrix}$$

$$\xi_{22} = \begin{bmatrix} \omega \\ h \\ 0 \end{bmatrix}$$

$$\xi_{23} = \begin{bmatrix} \frac{h}{2} \\ -\omega \\ 1 \end{bmatrix}$$

$$\xi_{31} = \begin{bmatrix} -\frac{h}{2} \\ 0 \\ 1 \end{bmatrix}$$

$$\xi_{32} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\xi_{33} = \begin{bmatrix} -\frac{h}{2} \\ -\omega \\ 1 \end{bmatrix}$$

Thus,

$$g_{st}(0) = \begin{bmatrix} I & \begin{bmatrix} \omega \\ 0 \end{bmatrix} \\ 0 & 1 \end{bmatrix}$$

Since in general (ω h) is not a unit vector, we have to normalize θ_{22} by the factor $\frac{1}{\sqrt{\omega^2+h^2}}$. Expanding the product of exponentials formula and simplifying, we yield three equations.

$$\Phi = (\theta_{11} + \theta 13) = (\theta_{21} + \theta_{23}) = (\theta_{31} + \theta_{33}) \tag{3}$$

where Φ is the angle the tool frame makes with the horizontal and

$$\begin{aligned} x &= (\omega + \theta_{12}) cos\theta_{11} + \frac{h}{2} sin(\theta_{11} + \theta_{13}) &(4)\\ &= \omega(1 + \theta_{22}) cos\theta_{21} - h(1 + \theta_{22}) sin\theta_{21} + \frac{h}{2} sin(\theta_{21} + \theta_{23}) &(5)\\ &= (\omega + \theta_{32}) cos\theta_{31} + \frac{h}{2} sin(\theta_{31} + \theta_{33}). &(6) \end{aligned}$$

where x is the x-coordinate of the of the origin of the tool frame. Finally,

$$\begin{aligned} y &= (\omega + \theta_{12}) sin\theta_{11} + \frac{h}{2} cos(\theta_{11} + \theta_{13}) + \frac{h}{2} &(7)\\ &= \omega(1 + \theta_{22}) sin\theta_{21} + h(1 + \theta_{22}) cos\theta_{21} - \frac{h}{2} cos(\theta_{21} + \theta_{23}) - \frac{h}{2} &(8)\\ &= (\omega + \theta_{32}) sin\theta_{31} + \frac{h}{2} cos(\theta_{31} + \theta_{33}) - \frac{h}{2} &(9) \end{aligned}$$

where y is the y-coordinate of the origin of the tool frame.

Part (c.):
We can relate d_1 and g_{st}:

$$x = d_1 cos\theta_{11} + \frac{h}{2} sin\phi \tag{10}$$

$$y = \frac{h}{2} - d_1 sin\theta_{11} + \frac{h}{2} cos\phi \tag{11}$$

Eliminating θ_{11}, we get

$$d_1 = sqrt(x - \frac{h}{s} sin\phi)^2 + (\frac{h}{2} - \frac{h}{2} cos\phi - y)^2 \tag{12}$$

Similarly,

$$x = d_2 cos(\theta_{21} + \theta) + \frac{h}{2} sin\phi \tag{13}$$

$$y = d_2 + h^2 sin(\theta_{21} + \theta) - \frac{h}{2} cos\phi - \frac{h}{2} \tag{14}$$

eliminating θ_{21} and θ, we get

$$d_2 = \sqrt{(x - \frac{h}{2} sin\phi)^2 + (y + \frac{h}{2} + \frac{h}{2} cos\phi)^2} \tag{15}$$

Finally,

$$x = d_3 cos\theta_{31} - \frac{h}{2} sin\phi \tag{16}$$

$$y = d_3 sin\theta_{31} + \frac{h}{s} cos\phi - \frac{h}{2} \tag{17}$$

Eliminating θ_{31}, we get

$$d_3 = \sqrt{(x + h sin\phi)^2 + (y + \frac{h}{2} - \frac{h}{2} cos\phi)^2} \tag{18}$$

Part (e.): The mechanism is singular if any of the following matrices lose rank

$$J_{s1} = \begin{bmatrix} \xi_{11} & \xi'_{12} & \xi'_{13} \end{bmatrix}$$

$$J_{s2} = \begin{bmatrix} \xi_{21} & \xi'_{22} & \xi'_{23} \end{bmatrix}$$

$$J_{s3} = \begin{bmatrix} \xi_{31} & \xi'_{32} & \xi'_{33} \end{bmatrix}$$

Now,

$$det(J_{s1}) = -(\omega + \theta_{12}) \tag{19}$$

$$det(J_{s2}) = -(\omega + \theta_{22}) \tag{20}$$

$$det(J_{s3}) = -(\omega + \theta_{32}) \tag{21}$$

Therefore, the mechanism is at a singular configuration if

$$d_i = (\omega + \theta_{i2}) = 0 \tag{22}$$

for any i. The mechanism is actuator singular if

$$\begin{bmatrix} \xi_{11} & \xi'_{13} & \xi_{21} & \xi'_{23} & \xi_{31} & \xi'_{33} \end{bmatrix} \begin{bmatrix} t\dot{het}a_{11} \\ \dot{\theta}_{13} \\ \dot{\theta}_{21} \\ \dot{\theta}_{23} \\ \dot{\theta}_{31} \\ \dot{\theta}_{33} \end{bmatrix} = \begin{bmatrix} \xi'_{12} & \xi'_{22} & \xi'_{32} \end{bmatrix} \begin{bmatrix} \dot{\theta}_{12} \\ \dot{\theta}_{22} \\ \dot{\theta}_{32} \end{bmatrix}$$

Problem 2:
Part (a.):

$$\xi_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\xi_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\xi_3 = \begin{bmatrix} l_1 \cos\theta_A \\ l_1 \sin\theta_A \\ 1 \end{bmatrix}$$

$$\xi_4 = \begin{bmatrix} \cos(\theta_A + \theta_C) \\ \sin(\theta_A + \theta_C) \\ 0 \end{bmatrix}$$

Part (b.):

$$Grammian = \begin{bmatrix} \xi_1 \cdot \xi_1 & \xi_1 \cdot \xi_2 & \xi_1 \cdot \xi_3 & \xi_1 \cdot \xi_4 \\ \xi_2 \cdot \xi_1 & \xi_2 \cdot \xi_2 & \xi_2 \cdot \xi_3 & \xi_2 \cdot \xi_4 \\ \xi_3 \cdot \xi_1 & \xi_3 \cdot \xi_2 & \xi_3 \cdot \xi_3 & \xi_3 \cdot \xi_4 \\ \xi_4 \cdot \xi_1 & \xi_4 \cdot \xi_2 & \xi_4 \cdot \xi_3 & \xi_4 \cdot \xi_4 \end{bmatrix}$$

To find the stationary configurations of joint B (labeled 2), we solve for when the determinant of the cofactor of the 2,2 element of the grammian is equal to zero.

$$0 = \det \begin{bmatrix} \xi_1 \cdot \xi_1 & \xi_1 \cdot \xi_3 & \xi_1 \cdot \xi_4 \\ \xi_3 \cdot \xi_1 & \xi_3 \cdot \xi_3 & \xi_3 \cdot \xi_4 \\ \xi_4 \cdot \xi_1 & \xi_4 \cdot \xi_3 & \xi_4 \cdot \xi_4 \end{bmatrix} \tag{23}$$

$$= \sin^2\theta_C + {l_1}^2 \tag{24}$$

Thus, for nonzero l_1, a stationary position occurs at θ_2=nπ.
Please note that there are other ways of describing the stationary position depending on how you define your angles.

Part (c.):
Simply repeat part (b), but for the determinant of the cofactor of the (1,1), (3,3), and (4,4) element.

Problem 4: MLS Chapter 5, problem (1.a)
No, this grasp is not force closure. It can not exert torques.

Problem 4: MLS Chapter 5, problem (2.a)
Yes. This grasp is force closure.