ME 115(b): Solution to Problem Set #4

Problem #1: Problem 2(b,d) of Chapter 5 in MLS text.
Part (b.)
Yes, this grasp is force closure.

Part (d.)
$B_1,\ B_2 = I.$

$$G_1 = \begin{bmatrix} I & S\begin{bmatrix}0\\-a\\b\end{bmatrix} \\ 0 & I \end{bmatrix} B_1$$

$$G_2 = \begin{bmatrix} I & S\begin{bmatrix}0\\a\\b\end{bmatrix} \\ 0 & I \end{bmatrix} B_1$$

The complete grasp matrix is

$$G = \begin{bmatrix} G_1 & G_2 \end{bmatrix}$$

Note that each G_i is full rank (since $B_i = I$).

Force-closure $\longleftrightarrow$ G is full rank and there exists $F_N \ \epsilon\ \eta(\mathrm{G})$ such that $F_N \ \epsilon$ FC. But G is full rank since each G_i is full rank. Furthermore, it has a null space of dimension 6 (nullity + rank =12) and since FC = $\Re^6 \times \Re^6$, *every* $F_N \ \epsilon\ \eta(\mathrm{G})$ is in FC. Therefore, force closure $\dim(\eta(\mathrm{G})) = 12 - rankG = 6$.

The grasp is *not* manipulable. The grasping constraint is

$$\begin{bmatrix} {B_1}^T & T_{C_1 S} & J_1 & 0 \\ 0 & {B_2}^T & T_{C_2 S} & J_2 \end{bmatrix} \dot{\theta} = G^T \begin{pmatrix} V_0 \\ \omega_0 \end{pmatrix} = \begin{bmatrix} {G_1}^T \\ {G_2}^T \end{bmatrix} \begin{pmatrix} V_0 \\ \omega_0 \end{pmatrix}$$

Since ${G_1}^T$ is full rank, we must have $\Re(\ {B_1}^T\ T_{C_1 S}\ J_1) \supseteq \Re({G_1}^T) = \Re^6$. But $J_1 \ \epsilon\ Re^{6\times 4} \longrightarrow$ its range has dimension 4 away from singularities, therefore the grasp is not manipulable.

Problem #2: Problem 3(b) of Chapter 5 in MLS text.

$$R_{C_1} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\hat{P}_{C_1} = \begin{bmatrix} 0 \\ -r \\ 0 \end{bmatrix}$$

$$\hat{P}_{C_1} = \begin{bmatrix} 0 & 0 & -r \\ 0 & 0 & 0 \\ r & 0 & 0 \end{bmatrix}$$

$$R_{C_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\hat{P}_{C_2} = \begin{bmatrix} 0 \\ r \\ 0 \end{bmatrix}$$

$$\hat{P}_{C_2} = \begin{bmatrix} 0 & 0 & r \\ 0 & 0 & 0 \\ -r & 0 & 0 \end{bmatrix}$$

Plugging these into equations for G_i:

$$G_1 = \begin{bmatrix} R_{C_1} & 0 \\ \hat{P}_{C_1} R_{C_1} & R_{C_1} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -r & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & r & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ -r & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & r & 0 & 0 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} R_{C_1} & 0 \\ \hat{P}_{C_1} R_{C_1} & R_{C_1} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & r & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ -r & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & r & 0 & 0 \\ 0 & 0 & 0 & -1 \\ -r & 0 & 0 & 0 \end{bmatrix}$$

Plugging G_is into the overall Grasp Map gives:

$$G = \begin{bmatrix} G_1 & G_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -r & 0 & 0 & 0 & 0 & r & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & r & 0 & 0 & -r & 0 & 0 & 0 \end{bmatrix}$$

Now to write the friction cone conditions:

$$FC = FC_{C_1} \times FC_{C_2} \tag{1}$$

$$FC_{C_1} = \{f_c : \sqrt{(f'_{C_1})^2 + (f_{C_1})^2} \leq \mu f^3{}_{C_1}, \|f^4{}_{C_1}\| \leq \gamma f^3{}_{C_1}, f^3{}_{C_1} \geq 0\} \tag{2}$$

$$FC_{C_2} = \{f_c : \sqrt{(f'_{C_2})^2 + (f_{C_2})^2} \leq \mu f^3{}_{C_2}, \|f^4{}_{C_2}\| \leq \gamma f^3{}_{C_2}, f^3{}_{C_2} \geq 0\} \tag{3}$$

The grasp map is full rank. Additionally, the 'force closure by inspection' method shown in class shows that this is a force closure grasp since it is a soft-finger grasp and a line between the two contact points can be drawn which is fully enclosed by both friction cones.

Problem #3: Problem 4(a) of Chapter 5 in MLS text.

We can derive the wrench basis by looking at which directions are free to movement. This contact model does not allow movement in the x,y, and z directions, or rotation along the x or z axis. It does, however, allow rotation along the y axis.

Thus the wrench basis is:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Friction cone:

$$\mu > 0 \tag{4}$$

$$\sqrt{f_x + f_y} \leq \mu \cdot f_n \tag{5}$$

$$f_n \geq 0 \tag{6}$$

$$|\tau_n| \leq \gamma_n f_n \tag{7}$$

$$|\tau_y| \leq \gamma_y f_y \tag{8}$$

Problem #4: Problem 5(a) of Chapter 5 in MLS text.

There are many possible answers to this problem. The easiest is to choose an equilateral triangle. You can then show geometrically that there is no way that a line between two point contacts can be made such that it will pass through both friction cones (in the case where μ = tan 30.

Problem #5: Problem 8(b) of Chapter 5 in MLS text.
We have an equilateral triangle and we wish to find all possible force-closure grasps for two contacts with friction and a coefficient of friction, μ=tan 45. The constraints that the line between contacts lie in the relevant friction cones become:

$$A : (\eta_1 - \mu{\eta_1}^{\perp}) \otimes (p_2 - p_1) > 0 \tag{9}$$
$$B : (\eta_1 + \mu{\eta_1}^{\perp}) \otimes (p_2 - p_1) < 0 \tag{10}$$
$$C : (\eta_2 - \mu{\eta_2}^{\perp}) \otimes (p_1 - p_2) > 0 \tag{11}$$
$$D : (\eta_2 + \mu{\eta_2}^{\perp}) \otimes (p_2 - p_1) < 0 \tag{12}$$

$$\eta_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{13}$$
$${\eta_1}^{\perp} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \tag{14}$$
$$\eta_2 = \begin{bmatrix} \cos 30 \\ -\sin 30 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} \\ -\frac{1}{2} \end{bmatrix} \tag{15}$$
$${\eta_2}^{\perp} = \begin{bmatrix} \sin 30 \\ \cos 30 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix} \tag{16}$$
$$p_1 = \begin{bmatrix} d_1 \\ 0 \end{bmatrix} \tag{17}$$
$$p_2 = \begin{bmatrix} d_2 \sin 30 \\ d_2 \cos 30 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} d_2 \\ \frac{\sqrt{3}}{2} d_2 \end{bmatrix} \tag{18}$$

$$\mu = \tan 45 = 1 \tag{19}$$

Plugging in these values into A,B,C and D:

$$A : d_1 + d_2 \frac{\sqrt{3} - 1}{2} > 0 \tag{20}$$
$$B : d_1 - d_2 \frac{\sqrt{3} - 1}{2} < 0 \tag{21}$$
$$C : -(\frac{\sqrt{3}}{2} - \frac{1}{2}) \frac{\sqrt{3}}{2} d_2 + (d_1 - \frac{d_2}{2})(\frac{1}{2} - \frac{\sqrt{3}}{2}) > 0 \tag{22}$$

$$D : -(\frac{\sqrt{3}}{2} + \frac{1}{2})\frac{\sqrt{3}}{2}d_2 - (d_1 - \frac{d_2}{2})(\frac{\sqrt{3}}{2} + \frac{1}{2}) < 0 \tag{23}$$

Problem #6:
Part #1: Yes, this grasp is force closure. We can use the visual method we learned last week, we can see that a line connecting the two point contacts clearly runs through both friction cones. **Part #2:**

$$R_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 0 \\ -d \\ 0 \end{bmatrix}$$

$$R_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 0 \\ d \\ 0 \end{bmatrix}$$

$$WrenchBasis = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} R_1 & 0 \\ \hat{P}_1 R_1 & R_1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ -d & 0 & 0 \\ 0 & 0 & 0 \\ 0 & d & 0 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} R_2 & 0 \\ \hat{P}_2 R_2 & R_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & d & 0 \\ 0 & 0 & 0 \\ -d & 0 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} G_1 & G_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ -d & 0 & 0 & 0 & d & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & d & 0 & -d & 0 & 0 \end{bmatrix}$$