ME 115(a): Homework #6 Solution

Problem 1: (15 points) Find the Denavit-Hartenberg parameters for manipulators (ii) and (iv) in Figure 3.23 of the MLS text.

Manipulator (ii): The choice of the stationary frame is abitrary. Place its origin along joint axis 1, but not necessarily at the point of coincidence of joint axes 1 and 2. The tool frame origin is placed in the middle of the "U", with its x-axis collinear with the mechanical link axis, and with its z-axis parallel to joint axis 2. In this case,

$$\begin{array}{llll} a_0 = 0 & \alpha_0 = 0 & d_1 \neq 0 & \theta_1 = \text{ variable} \\ a_1 = 0 & \alpha_1 = \frac{\pi}{2} & d_2 = 0 & \theta_2 = \text{ variable} \\ a_2 \neq 0 & \alpha_2 = -\frac{\pi}{2} & d_3 \neq 0 & \theta_3 = \text{ variable} \\ a_3 \neq 0 & \alpha_3 = \frac{\pi}{2} & d_4 = 0 & \theta_4 = \text{ constant} \end{array}$$

where the value of d_1 will be determined by the location of the stationary frame origin.

Manipulator (iv): The choice of the stationary frame is abitrary. For simplicity, place the origin of the stationary frame at the point where all three joints intersect. Place the z-axis of the stationary frame, z_S, collinear with the first joint axis. Orient the x-axis of the stationary frame to be orthogonal to both joint axes 1 and 2 (pointing toward the right in Figure 3.23(i)). Similary, there are many choices for the tool frame. Let's assume that the tool frame is parallel with the link frame of link 3, (as determined using the Denavit-Hartenberg procedure), but its origin lies at the tip of the mechanism (in the "U" of Figure 3.23(iv)). Then, the D-H parameters are:

$$\begin{array}{llll} a_0 = 0 & \alpha_0 = 0 & d_1 = 0 & \theta_1 = \text{ variable} \\ a_1 = 0 & \alpha_1 = -\frac{\pi}{2} & d_2 = 0 & \theta_2 = \text{ variable} \\ a_2 = 0 & \alpha_2 = \frac{\pi}{2} & d_3 = \text{variable} & \theta_3 = \ 0 \\ a_3 = 0 & \alpha_3 = 0 & d_4 = \text{constant} & \theta_4 = \text{ constant} = 0 \end{array}$$

where the constant d_4 depends upon the offset between the origin of link frame 3 and the origin of the tool frame.

Problem 2: (20 points) Consider the simple manipulator (iii) associated with Prob.4 in Chapter 3 of the MLS text.

- **Part (a):** *Determine the Denavit-Hartenberg parameters of this manipulator.*

  For simplicity, let us choose the z-axis of the stationary frame to be collinear with the first joint axis. The origin of the stationary frame is located some distance below the point of intersection of the first two axes. Also, choose the tool frame origin to coincide with the intersection point of the last three axes (the "wrist"). Also, assume that the link 6 frame of the Denavit-Hartenberg approach is the tool frame.

$$\begin{array}{llll}
a_0 = 0 & \alpha_0 = 0 & d_1 \neq 0 & \theta_1 = \text{ variable} \\
a_1 = 0 & \alpha_1 = \frac{\pi}{2} & d_2 = 0 & \theta_2 = \text{ variable} \\
a_2 = 0 & \alpha_2 = -\frac{\pi}{2} & d_3 = \text{variable} & \theta_3 = 0 \text{ (constant)} \\
a_3 = 0 & \alpha_3 = \frac{\pi}{2} & d_4 = 0 & \theta_4 = \text{ variable} \\
a_4 = 0 & \alpha_4 = \frac{\pi}{2} & d_5 = 0 & \theta_5 = \text{ variable} \\
a_5 = 0 & \alpha_5 = \frac{\pi}{2} & d_6 = 0 & \theta_6 = \text{ variable} \\
a_6 = 0 & \alpha_6 = 0 & d_7 = 0 & \theta_7 = 0
\end{array}$$

- **Part (b):** *Find the forward kinematics using the Denavit-Hartenberg approach.* To find the forward kinematics using the Denavit-Hartenberg approach, one must use the formula

$$g_{ST}(\vec{\theta}) = g_{S1}(\theta_1)g_{12}(\theta_2)g_{23}(d_3)g_{34}(\theta_4)g_{45}(\theta_5)g_{56}(\theta_6)g_{6T}.$$

where each $g_{i,i+1}$ is given by:

$$\begin{bmatrix} \cos\theta_{i+1} & -\sin\theta_{i+1} & 0 & a_i \\ \sin\theta_{i+1}\cos\alpha_i & \cos\theta_{i+1}\cos\alpha_i & -\sin\alpha_i & -d_{i+1}\sin\alpha_i \\ \sin\theta_{i+1}\sin\alpha_i & \cos\theta_{i+1}\sin\alpha_i & \cos\alpha_i & d_{i+1}\cos\alpha_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Plugging in the D-H parameters from above yields:

$$g_{ST} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -\sin\theta_2 & -\cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_5 & \cos\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} g_{6T}$$

- **Part (c):** *Find the forward kinematics using the Product-of-Exponentials Approach.* This part of the problem was cancelled