## ME 115(a) 2006: Solution to Homework #5

**Problem** #1: Each finger applies a "wrench" to the disk object due to its contact with the disk. Since we are assuming a frictionless contact, the finger can only apply forces to the disk that are normal to the disk's boundary. Hence, each finger applies a pure force in the direction of the boundary normal vector, which corresponds to a zero pitch screw.

Define a coordinate system whose origin lies at the common intersection of all of the finger forces at the center of the disk. Choose the z-axis of this system to be normal to the plane of the disk. Let the x-axis coincide with one of the finger contact normals. Thus, the screw coordinates for the three wrenches are:

$$\xi_1 = \begin{bmatrix} -1\\0\\0\\0\\0\\0\\0 \end{bmatrix} \qquad \xi_2 = \begin{bmatrix} -\cos(120^o)\\-\sin(120^o)\\0\\0\\0\\0 \end{bmatrix} \qquad \xi_2 = \begin{bmatrix} -\cos(240^o)\\-\sin(240^o)\\0\\0\\0\\0 \end{bmatrix}$$

If the disk is not immobilized, there there must exist a twist (i.e., an instantaneous motion of the disk) that is **reciprocal** to the finger wrenches. The  $\xi_R = [0 \ 0 \ 1 \ 0 \ 0 \ 0]^T$  denote the zero pitch twist that corresponds to rotation of the disk about a vertical axis passing through the origin of the reference frame (i.e., the concurrency point of the three contact normals). This twist is reciprocal to each of the finger wrenches, and therefore the fingers **can not** stop any rotational motions of the disk. Hence, the object is not immobilized.

**Problem** #3: Find the Denavit-Hartenberg parameters for manipulators (i) (ii) and (iv) in Figure 3.23 of the MLS text

• (i) The choice of the stationary frame is abitrary. For simplicity, place the origin of the stationary frame at the point where all three revolute joints intersect. Place the z-axis of the stationary frame,  $z_S$ , collinear with the first joint axis. Orient the x-axis of the stationary frame to be orthogonal to both joint axes 1 and 2 (pointing toward the right in Figure 3.23(i)). Similary, there are many choices for the tool frame. Let's assume that the tool frame is coincident with the link frame of link 3, as determined using the Denavit-Hartenberg procedure. Then, the D-H parameters are:

$$\begin{array}{rrrr} a_{0} = 0 & \alpha_{0} = 0 & d_{1} = 0 & \theta_{1} = \text{ variable} \\ a_{1} = 0 & \alpha_{1} = -\frac{\pi}{2} & d_{2} = 0 & \theta_{2} = \text{ variable} \\ a_{2} = 0 & \alpha_{2} = \frac{\pi}{2} & d_{3} = 0 & \theta_{3} = \text{ variable} \\ a_{3} = 0 & \alpha_{3} = 0 & d_{4} = 0 & \theta_{4} = \text{ constant} = 0 \end{array}$$

• (ii) The choice of the stationary frame is abitrary. Place its origin along joint axis 1,

but not necessarily at the point of coincidence of joint axes 1 and 2. The tool frame origin is placed in the middle of the "U", with its x-axis collinear with the mechanical link axis, and with its z-axis parallel to joint axis 2. In this case,

$$\begin{array}{lll} a_0 = 0 & \alpha_0 = 0 & d_1 \neq 0 & \theta_1 = \text{ variable} \\ a_1 = 0 & \alpha_1 = \frac{\pi}{2} & d_2 = 0 & \theta_2 = \text{ variable} \\ a_2 \neq 0 & \alpha_2 = -\frac{\pi}{2} & d_3 \neq 0 & \theta_3 = \text{ variable} \\ a_3 \neq 0 & \alpha_3 = \frac{\pi}{2} & d_4 = 0 & \theta_3 = \text{ constant} \end{array}$$

where the value of  $d_1$  will be determined by the location of the stationary frame origin.

• (iv) The choice of the stationary frame is abitrary. For simplicity, place the origin of the stationary frame at the point where all three joints intersect. Place the z-axis of the stationary frame,  $z_S$ , collinear with the first joint axis. Orient the x-axis of the stationary frame to be orthogonal to both joint axes 1 and 2 (pointing toward the right in Figure 3.23(i)). Similary, there are many choices for the tool frame. Let's assume that the tool frame is parallel with the link frame of link 3, (as determined using the Denavit-Hartenberg procedure), but its origin lies at the tip of the mechanism (in the "U" of Figure 3.23(iv)). Then, the D-H parameters are:

where the constant  $d_4$  depends upon the offset between the origin of link frame 3 and the origin of the tool frame.