CDS 110(b): Winter 2011/2012

(due Monday, January 15, 2012) [1]

Problem #1: In class we derived the optimal bang-bang control feedback for a system consisting of a unit mass bead pushed along a wire by thrust u. The dynamical equation governing this system is::

$$\ddot{x} = u$$

where the control input u is contstrained as: $|u| \leq 1$. Rederive the optimal feedback law for the case when the dynamical system is governed by the equation $m\ddot{x} = u$ (where m is the mass of the bead on the wire) and u is constrained as: $|u| \leq F$.

Problem #2: Use the maximum principle to show that the shortest path between two points in the plane is a straight line. Model a control system

$$\dot{x} = u$$

where $x \in \mathbb{R}^2$ is the position of points in the plane and $u \in \mathbb{R}^2$ is the velocity of a point along a curve. Find the path of minimal length connecting $x(0) = x_0$ and $x(1) = x_f$. To minimize the length of the curve, let the cost along the path be

$$J = \int_0^1 ||\dot{x}||\, dt = \int_0^1 \sqrt{\dot{x}^T \dot{x}}\, dt$$

subject to the initial and final position constraints.

Problem #3: Consider again the "double integrator" model

$$\ddot{x} = u\ .$$

In this problem, we wish to minimize the *energy* used by the system in going from $x(0) = 1$ to $x(1) = 0$. Assume that the unit mass particle has initial velocity $\dot{x}(0) = 1$, and the final desired velocity is $\dot{x}(1) = 0$ (i.e., bring the object to rest at the origin). The goal is to minimize the cost function:

$$\min_{u(t)} J = \int_0^1 u^2(t)\, dt$$

for the given initial and final conditions.

- find an expression for $u(t)$
- plot $u(t)$, $x(t)$, and $\dot{x}(t)$ over the time interval $t \in [0, 1]$

[1] Please put the number of hours you spent on this homework (including reading) in the upper left-hand corner of the first page of your homework