# CDS 110(b): Winter 2011/2012

(due Wednesday, Feb 15, 2012)

These two problems will have you implement a simple finite horizon receding horizon optimal control problem (Problem 2), and compare it to an infinite horizon optimal solution (Problem 1) .

**Problem #1:** Consider a system whose dynamics are governed by the equations:

$$\dot{x} \;=\; ax \;+\; bu \tag{1}$$

where $x \in \mathbb{R}$ denotes the state, $u \in \mathbb{R}$ is a single scalar control input, and $a$, $b$ are constant, *positive* scalars. Consider the optimal control problem with cost

$$J \;=\; \frac{1}{2}\int_{t_0}^{T} u^2(t)\; dt \;+\; \frac{1}{2}c\; x^2(T) \tag{2}$$

where final time $T$ is given, and $c > 0$ is a constant. The optimal control for finite time $T > 0$ is derived in Example 2.2 of the optimal control notes. Now consider the infinite horizon problem with infinite horizon cost:

$$J \;=\; \frac{1}{2}\int_{t_0}^{\infty} u^2(t)\; dt \tag{3}$$

**part (a):** Solve the algebraic Riccati equation to find $P$, leading to the optimal control $u^*(t) \;=\; -bPx^*(t)$.

**part (b):** Plot the state solution of the finite optimal controller for the following parameter values

$$a = 2 \qquad b = 0.5 \qquad x(t_0) = 4c = 0.1, 10.0 \quad t_f = 0.5, 1, 10 \tag{4}$$

Compare these to the infinite time optimal control solution. Which finite time solution is the closest to the infinite time solution?

**Problem #2:** Using the solution given in Equation (2.5) of the optimal control notes (Chapter 2), implement (for the dynamical system of Problem 1) a finite-horizon optimal controller in a receding horizon fashion with a horizon time of $T = 0.5$ units. Using the parameter values in part (b) of Problem 1, compare the responses of the receding horizon controllers to the LQR controller designed in Problem 1, using the same initial condition.

*Note:* Since Example 2.2 of the text gives you a closed form formula for the optimal control and for the state of the of the system under the influence of that optimal control, you can proceed as follows:

1. set $t_0 = 0$.

2. using the closed form solution for the optimal state, plot $x(t)$ over the horizon time, and then save $x_T = x(t_0 + T)$.

3. set $x(t_0) = x_T$, and repeat step 2 until $x$ is small.