CALIFORNIA INSTITUTE OF TECHNOLOGY

Control and Dynamical Systems

CDS 110b

Problem Set #4

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Problem 1: Kalman filters are often an integral part of on-board vehicle navigation systems. Many on-board navigation systems use an *Inertial Measurement Unit* (IMU), which provides (noisy) measurements of the vehicle's acceleration, as well as gyroscopes to provide (noisy) measurements of the vehicle's rate of rotation. The IMU measurements can be coupled with those of a GPS receiver to provide high rate updates on vehicle position between GPS measurements, and to smooth out GPS measurement errors . Particularly during GPS *blackouts* the IMU provides the only vehicle position reference, which may be essential for vehicle navigation.

In this highly simplified version of the navigation problem, we will assume that a vehicle whose motions are restricted to the x-axis contains an accelerometer which can measure acceleration along that axis. Let *x* denote the vehicle position. The vehicle dynamics are given by:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} = u$$

Where *m* is the vehicle mass, *b* is the vehicle damping coefficient (accounting, e.g., for wind resistance), and *u* is the input (in units of force) to the vehicle. For this problem, let m=1000 kg, and b/m=0.03 s^{-1}

- **Part (a):** develop a discrete time dynamical equation of motion for this system
- **Part (b):** assume that the IMU can measure vehicle acceleration at a rate of 20 Hz, with an accuracy of 0.0981 m/s^2 (which is a 1% measurement accuracy). Further assume that GPS measurements are received at a rate of 1 Hz, with an accuracy of 5.0 meters. Assume that both measurement uncertainties can be modeled as zero mean Gaussian distributed white noise with std. deviation equal to the stated accuracies. Assuming there are no external disturbance forces acting on the vehicle, compute the time history of the estimation covariance, and plot the covariance associated with the positional portion of the covariance for the first 20 seconds of vehicle motion. Assume that at the beginning of the motion, a GPS measurement of vehicle position is available and an IMU measurement of vehicle acceleration is available.

Problem 2: Fixed lag smoothing can improve the estimate of vehicle position at the cost of delay in making that information available. For the same system described in Problem #1, find the fixed lag smoothing equations for a smoother which delays the position estimate by 2 seconds (e.g., by two delays in the GPS measurement cycle). What is the covariance improvement of the fixed lag smoother over that of the Kalman filter?

Problem 3: We derived the form of the discrete time Kalman filter under the assumption that at all times t_k the process noise, η_k, and the measurement noise, ω_k, are independent. However, in some cases these disturbances may be correlated. Assume that η_k and ω_k are zero mean, Gaussian, white discrete time noise processes with the following properties:

$$E\begin{bmatrix}\eta_k \\ \omega\end{bmatrix}=\begin{bmatrix}0 \\ 0\end{bmatrix}; \qquad E\left(\begin{bmatrix}\eta_k \\ \omega_k\end{bmatrix}\begin{bmatrix}\eta_l^T & \omega_l^T\end{bmatrix}\right)=\begin{bmatrix}Q & S \\ S^T & R\end{bmatrix}\delta_{kl}$$

where E is the expectation operator, and δ_{kl} is the dirac delta function: $\delta_{kl} = 1$ when k=l, $\delta_{kl} = 0$ otherwise. Derive the form of the discrete time state estimate mean and covariance update equations under these conditions.